Problem Set 1
Introduction to Modal Logic

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1. **Exercise 1.3.2: (a), (b), (d) and (g), pg. 27:** Let $\mathfrak{N} = \langle \mathbb{N}, S_1, S_2 \rangle$ and $\mathfrak{B} = \langle \mathbb{B}, R_1, R_2 \rangle$ be the following frames for a modal similarity type with two diamonds $\Diamond_1$ and $\Diamond_2$. Here, $\mathbb{N}$ is the set of natural numbers, $\mathbb{B}$ is the set of stirngs of 0s and 1s, and the relations are defined by

$$\begin{array}{rll} mS_1n & \text{iff} & n = m + 1 \\ mS_2n & \text{iff} & m > n \\ sR_1t & \text{iff} & t = s0 \text{ or } t = s1 \\ sR_2t & \text{iff} & t \text{ is a proper initial segment of } s \end{array}$$

   Which of the folloiwng formulas are valid on $\mathfrak{N}$, $\mathfrak{B}$, respectively?

   (a) $(\Diamond_1 p \wedge \Diamond_1 q) \to \Diamond_1(p \wedge q)$

   (b) $(\Diamond_2 p \wedge \Diamond_2 q) \to \Diamond_2(p \wedge q)$

   (c) $p \to \Diamond_1 \Box_2 p$

   (d) $p \to \Box_2 \Diamond_1 p$

2. **Exercise 1.3.5, (a), (b), and (d), pg. 27:** Show that each of the following formulas is *not* valid by constructing a frame $\mathfrak{F} = \langle W, R \rangle$ that refutes it.

   (a) $\Box\bot$

   (b) $\Diamond p \to \Box p$

   (c) $\Diamond\Box p \to \Box\Diamond p$

3. **Exercise 1.6.1, pg. 37:** Give **K**-proofs (i.e., **K** derivations) of $(\Box p \wedge \Diamond q) \to \Diamond(p \wedge q)$ and $\Diamond(p \vee q) \leftrightarrow (\Diamond p \vee \Diamond q)$.

4. **Exercise 1.6.7, pg. 37:** Let F be a class of frames. Show that Λ_{F} is a normal modal logic. Some definitions to remember:

- $\Vdash_{\mathsf{F}} \phi$ iff $\mathfrak{F} \Vdash \phi$ for each $\mathfrak{F} \in \mathsf{F}$.
- $\Lambda_{\mathsf{F}} = \{\phi \mid \ \Vdash_{\mathsf{F}} \phi\}$
- A logic L is a **normal modal logic** if it contains all propositional tautologies, the axioms *Dual* and K, and is closed under the rules, *uniform substitution*, *modus ponens* and *generalization*.