**Problem Set 2**
**Introduction to Modal Logic**
Institute for Logic, Language and Computation
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**Exercise 2.1.1, p. 63** Suppose we wanted an operator $D$ with the following satisfaction definition: for any model $\mathfrak{M}$ and any formula $\phi$, $\mathfrak{M}, w \Vdash \phi$ iff there is a $u \neq w$ such that $\mathfrak{M}, u \Vdash \phi$. This operator is called the *difference operator*. Is the difference operator definable in the basic modal language?

**Exercise 2.1.3, p. 63** Give the simples possible example which shows that the truth of modal formulas is *not* invariant under homomorphisms, even if condition 1 is strengthened to an equivalence. Is modal truth preserved under homomorphisms? [Condition 1 for $f : \mathfrak{M} \to \mathfrak{M}'$ to be a homomorphism is: for each proposition letter $p$ and each element $w$ from $\mathfrak{M}$, if $w \in V(p)$, then $f(w) \in V'(p)$.]

**Exercise 2.1.4, p. 63** Show that the mapping defined in the proposition 2.15 is indeed a surjective bounded homomorphism. [You should remember the method presented in the previous lecture for unwinding a model $\mathfrak{M}$ into a rooted tree-like model $\mathfrak{M}'$, with root $w$. The mapping the exercise refers to is the mapping $f : \mathfrak{M} \to \mathfrak{M}'$ where $f(w, u_1, \ldots, u_n) \mapsto u_n$.]