Problem Set 4
Introduction to Modal Logic

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1. Give a concrete example of a model $\mathfrak{M}$ and a filtration $\mathfrak{M}^f$ such that $\mathfrak{M}$ is transitive but $\mathfrak{M}^f$ is not.

2. Prove Lemma 2.42 on pg. 81: Let $\mathfrak{M}$ be a model, Σ a subformula closed set of formulas, and W_Σ the set of equivalence classes induced on $\mathfrak{M}$ by $\leftrightsquigarrow_\Sigma$. Let R^t be the binary relation on W_Σ defined by:

$$|w|R^t|v| \text{ iff for all } \phi, \text{ if } \Diamond\phi \in \Sigma \text{ and } \mathfrak{M}, v \Vdash \phi \vee \Diamond\phi \text{ then } \mathfrak{M}, w \Vdash \Diamond\phi$$

3. Suppose that we work in a modal language containing only a finite number of propositional variables. Let $\mathfrak{M}$ and $\mathfrak{M}'$ be models for this language. Prove that for every $w \in W$ and $w' \in W'$, if w and w' agree on all formulas of degree at most n, then w and w' are n-bisimular.