Problem Set 5
Introduction to Modal Logic

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1. Is each one of the formulas P_1, P_2 and P_3 given below equivalent to the standard translation of some modal formula (in the basic modal language)? If you say yes, show a concrete formula (and its translation) justifying your claim. If you say no, give a short but rigorous proof.

   (a) $\forall y(\neg xRy)$

   (b) $\forall y(xRy \wedge yRx)$

   (c) $\exists y(xRy \wedge \exists z(yRz \wedge P_1(x) \wedge P_2(z)))$

2. Let Φ be a sete of atomic propositions. A formula of $\mathcal{L}^*$ has the following syntactic form:
$$\phi := p \mid \bot \mid \neg\phi \mid \phi \vee \psi \mid \Diamond\phi \mid \Diamond^*\phi$$

   Models for $\mathcal{L}^*$ are interpreted in standard Kripke models $\mathfrak{M} = \langle W, R, V\rangle$. Truth is defined as usual. We only give the definition of truth of $\Diamond^*\phi$:
$$\mathfrak{M}, w \Vdash \Diamond^*\phi \text{ iff } \exists v,\ wR^*v \text{ and } \mathfrak{M}, v \Vdash \phi$$

   where R^* is the reflexive transitive closure of R. Prove that $\mathcal{L}^*$ satisfies the Finite Model Property. **Hint:** Given a formula ϕ, consider the set of formulas $CL(\phi)$, the smallest subformulas closed set containing ϕ and such that if $\Diamond^*\psi \in CL(\phi)$ then $\Diamond\Diamond^*\psi \in CL(\phi)$

3. Let Z be a bisimulation that links $\mathfrak{M} = \langle W, \{R_a \mid a \in P\}, V\rangle$ and $\mathfrak{M}' = \langle W', \{R'_a \mid a \in P\}, V'\rangle$, where P is a set of atomic programs. Show that Z is a bisimulation for test-free PDL (PDL without the test operator). That is, show Z is a bisimulation linking $\mathfrak{M} = \langle W, \{R_\alpha\}, V\rangle$ and $\mathfrak{M}' = \langle W', \{R'_\alpha\}, V'\rangle$, where α is a program generated by the set of atomic programs P.