Bonus Problem Set 1
Model Theory

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1. Prove that for the first order language L, there are at most $2^{|L|}$ nonequivalent models (this is essentially question 2 on page 5). Find necessary and sufficient conditions on L so that there will be *exactly* $2^{|L|}$ non-equivalent models. In general, find find the conditions on L so that there are exactly 2^κ non-equivalent models for each infinite cardinal κ.

2. Let A be an L-structure and consider the first-order language $(L_{\omega,\omega})$. Call an element of A **definable** if there is a formula of $L_{\omega,\omega}$ such that a is the only element of A satisfying ϕ. For each $n \in \omega$, find a model a language with only a finite number of symbols and a model A_n for this language which has exactly n undefinable elements. (For $n = 0$ and $n > 1$ the answer is easy. It is hard for $n = 1$.)