Problem Set 1
Recursion Theory

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[CT] is *Computability Theory* by Barry Cooper.

1. Show that the following functions are primitive recursive:

   (a) $f(n) = n!$, where $n!$ is defined as

   $$n! = \begin{cases} 1 \times 2 \times 3 \times \cdots \times (n-1) \times n & n > 0 \\ 1 & n = 0 \end{cases}$$

   (b) $f(n_1, n_2, \ldots, n_k) = \min\{n_1, \ldots, n_k\}$ (the least of the number $n_1, \ldots, n_k$)

2. Exercise 2.1.13, pg. 16 of [CT]

3. Let A be the Ackermann function defined by (see pg. 17 of [CT])

   $$\begin{aligned} A(m,0) &= m+1 \\ A(0,n+1) &= A(1,n) \\ A(m+1,n+1) &= A(A(m,n+1),n) \end{aligned}$$

   Prove

   (a) $A(m,3) = 2^{m+3} - 3$

   (b) A is a total function