

# Lecture 1: Introduction to Social Software

Eric Pacuit

ILLC, University of Amsterdam

[staff.science.uva.nl/~epacuit](http://staff.science.uva.nl/~epacuit)

[epacuit@science.uva.nl](mailto:epacuit@science.uva.nl)

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Caput Logic, Language and Information: Social Software

[staff.science.uva.nl/~epacuit/caputLLI.html](http://staff.science.uva.nl/~epacuit/caputLLI.html)

## Course Information

**Time & Place:** Thursdays, 13 - 15, Room P.019

**Course Material:** There is no textbook for the course. Students will be required to read numerous research articles.

**Content:** This will be discussed today!

**Evaluation:** Some problem sets, a paper, and (possibly) a short presentation. Details will follow.

**Website:** [staff.science.uva.nl/~epacuit/caputLLI.html](http://staff.science.uva.nl/~epacuit/caputLLI.html)

**Email:** [epacuit@staff.science.uva.nl](mailto:epacuit@staff.science.uva.nl)

## Introduction: What is social software?

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- A Theory of Correctness of Social Procedures

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- Mathematical Models of Social Situations
- A Theory of Correctness of Social Procedures
- Designing Social Procedures

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## Related Fields:

- Computer Science (algorithms, logic of programs, ...),
- Philosophical Logic (epistemic logic, deontic logic, ...),
- Game Theory/Decision Theory (mechanism design, ...),
- Social Choice (computational social choice, ...),
- Artificial Intelligence (multi-agent systems, ...)

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Just as computer programs have logical properties which are analyzed by means of appropriate logics of programs, social procedures also have logical properties which can be analyzed with the appropriate logical tools.

- Game Logic (Parikh, Pauly)
- Coalitional Logic (Pauly, Goranko)
- Game Algebra (van Benthem, Goranko, Venema, Pauly)

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### Social procedures occur at two levels

1. Individual algorithms set up by a group of agents  
(taking a train, dropping a class, ...)
2. Algorithms that require a group even in the execution  
(voting procedures, fair division algorithms, piano duets, ...)

## Social Software: Further Reading

- *Social Software*, Rohit Parikh, Synthese 132, 2002.
- *Language as Social Software*, Rohit Parikh, 2001.
- *Social Interaction, Knowledge, and Social Software*, EP and Rohit Parikh, forthcoming in *Interactive Computation: The New Paradigm*
- *Logic for Mechanism Design – A Manifesto*, Marc Pauly and Michael Wooldridge, unpublished manuscript.
- Chapter 1 of *Topics in Social Software: Information in Strategic Situations*, EP, 2005.

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## Homework

- *Social Software*, Rohit Parikh, *Synthese* 132, 2002.
- *Language as Social Software*, Rohit Parikh, 2001.
- *Social Interaction, Knowledge, and Social Software*, EP and Rohit Parikh, forthcoming in *Interactive Computation: The New Paradigm*
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## Social Software: Examples

There are two well-studied areas of social software

1. Voting Theory: *Voting Procedures* by Steven Brams and Peter Fishburn, *Basic Geometry of Voting* by Donald Saari
2. Fair Division

## Adjusted Winner

**Adjusted winner (AW)** is an algorithm for dividing  $n$  divisible goods among two people (invented by Steven Brams and Allan Taylor).

For more information see

- *Fair Division: From cake-cutting to dispute resolution* by Brams and Taylor, 1998
- *The Win-Win Solution* by Brams and Taylor, 2000
- [www.nyu.edu/projects/adjustedwinner](http://www.nyu.edu/projects/adjustedwinner)

## Adjusted Winner: Example

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Item	Ann	Bob
$A$	5	4
$B$	65	46
$C$	30	50
<b>Total</b>	100	100

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Suppose Ann and Bob are dividing three goods:  $A$ ,  $B$ , and  $C$ .

**Step 2.** The agent who assigns the most points receives the item.

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Item	Ann	Bob
$A$	5	0
$B$	65	0
$C$	0	50
<b>Total</b>	70	50

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Notice that  $65/46 \geq 5/4 \geq 1 \geq 30/50$

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Item	Ann	Bob
$A$	0	4
$B$	65	0
$C$	0	50
<b>Total</b>	<b>65</b>	<b>54</b>

## Adjusted Winner: Example

Suppose Ann and Bob are dividing three goods:  $A$ ,  $B$ , and  $C$ .

**Step 3.** Equitability adjustment:

Still not equal, so give (some of)  $B$  to Bob:  $65p = 100 - 46p$ .

Item	Ann	Bob
$A$	0	4
$B$	65	0
$C$	0	50
<b>Total</b>	<b>65</b>	<b>54</b>

## Adjusted Winner: Example

Suppose Ann and Bob are dividing three goods:  $A$ ,  $B$ , and  $C$ .

**Step 3.** Equitability adjustment:

$$\text{yielding } p = 100/111 = 0.9009$$

Item	Ann	Bob
$A$	0	4
$B$	65	0
$C$	0	50
<b>Total</b>	<b>65</b>	<b>54</b>

## Adjusted Winner: Example

Suppose Ann and Bob are dividing three goods:  $A$ ,  $B$ , and  $C$ .

**Step 3.** Equitability adjustment:

$$\text{yielding } p = 100/111 = 0.9009$$

Item	Ann	Bob
$A$	0	4
$B$	58.559	4.559
$C$	0	50
<b>Total</b>	<b>58.559</b>	<b>58.559</b>

## Adjusted Winner: Formal Definition

Suppose that  $G_1, \dots, G_n$  is a fixed set of goods.

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Suppose that  $G_1, \dots, G_n$  is a fixed set of goods.

A valuation of these goods is a vector of natural numbers  $\langle a_1, \dots, a_n \rangle$  whose sum is 100.

Let  $\alpha, \alpha', \alpha'', \dots$  denote possible valuations for Ann and  $\beta, \beta', \beta'', \dots$  denote possible valuations for Bob.

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An allocation is a vector of  $n$  real numbers where each component is between 0 and 1 (inclusive). An allocation  $\sigma = \langle s_1, \dots, s_n \rangle$  is interpreted as follows.

For each  $i = 1, \dots, n$ ,  $s_i$  is the proportion of  $G_i$  given to Ann.

Thus if there are three goods, then  $\langle 1, 0.5, 0 \rangle$  means, “Give all of item 1 and half of item 2 to Ann and all of item 3 and half of item 2 to Bob.”

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Suppose that  $G_1, \dots, G_n$  is a fixed set of goods.

$V_A(\alpha, \sigma) = \sum_{i=1}^n a_i s_i$  is the total number of points that Ann receives.

$V_B(\beta, \sigma) = \sum_{i=1}^n b_i(1 - s_i)$  is the total number of points that Bob receives.

Thus  $AW$  can be viewed as a function from pairs of valuations to allocations:  $AW(\alpha, \beta) = \sigma$  if  $\sigma$  is the allocation produced by the  $AW$  algorithm.

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$\text{AW}(\alpha, \beta)$  is defined as follows:

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1. Give all the goods  $G_1, \dots, G_r$  to Ann and  $G_{r+1}, \dots, G_n$  to Bob. Let  $X, Y$  be the number of points received by Ann and Bob respectively. Assume for simplicity that  $X \geq Y$ .

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1. Give all the goods  $G_1, \dots, G_r$  to Ann and  $G_{r+1}, \dots, G_n$  to Bob. Let  $X, Y$  be the number of points received by Ann and Bob respectively. Assume for simplicity that  $X \geq Y$ .
2. If  $X = Y$ , then stop. Otherwise, transfer a portion of  $G_r$  from Ann to Bob which makes  $X = Y$ . If equitability is not achieved even with all of  $G_r$  going to Bob, transfer  $G_{r-1}, G_{r-2}, \dots, G_1$  in that order to Bob until equitability is achieved.

## Fairness Conditions

- **Proportional** if both Ann and Bob receive at least 50% of their valuation. That is,  $\sum_{i=1}^n s_i a_i \geq 50$  and  $\sum_{i=1}^n (1 - s_i) b_i \geq 50$
- **Envy-Free** if no party is willing to give up its allocation in exchange for the other player's allocation. That is,  $\sum_{i=1}^n s_1 a_i \geq \sum_{i=1}^n (1 - s_i) a_i$  and  $\sum_{i=1}^n (1 - s_i) b_i \geq \sum_{i=1}^n s_i b_i$ .
- **Equitable** if both players receive the same total number of points. That is  $\sum_{i=1}^n s_i a_i = \sum_{i=1}^n (1 - s_i) b_i$
- **Efficient** if there is no other allocation that is strictly better for one party without being worse for another party. That is for each allocation  $\sigma' = \langle s'_1, \dots, s'_n \rangle$  if  $\sum_{i=1}^n a_i s'_i > \sum_{i=1}^n a_i s_i$ , then  $\sum_{i=1}^n (1 - s'_i) b_i < \sum_{i=1}^n (1 - s_i) b_i$ . (Similarly for Bob).

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## Easy Observations

- For two-party disputes, proportionality and envy-freeness are equivalent.
- $AW$  only produces equitable allocations (equitability is essentially built in to the procedure).
- $AW$  produces allocations  $\sigma$  that have the following property: there is at most one  $i$  such that  $0 \leq \sigma_i \leq 1$  and for all  $j \neq i$ ,  $\sigma_j \in \{0, 1\}$ .

## *Adjusted Winner is Fair*

Theorem (Brams and Taylor, 1995) AW produces allocations that are efficient, equitable and envy-free (with respect to the announced valuations)

## **Social Software: Important Issues**

The agents *knowledge/beliefs/state of information* plays an important role in the analysis of social software.

## **Example I**

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

## Example I

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There is a very simple procedure to solve Ann's problem: *have a (trusted) friend tell Bob the time and subject of her talk.*

## Example I

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

There is a very simple procedure to solve Ann's problem: *have a (trusted) friend tell Bob the time and subject of her talk.*

Taking a cue from computer science, we can ask is this procedure correct?

## **Example I**

Yes, if

1. Ann knows about the talk.

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Yes, if

1. Ann knows about the talk.
2. Bob knows about the talk.

## Example I

Yes, if

1. Ann knows about the talk.
2. Bob knows about the talk.
3. Ann knows that Bob knows about the talk.

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Yes, if

1. Ann knows about the talk.
2. Bob knows about the talk.
3. Ann knows that Bob knows about the talk.
4. Bob *does not* know that Ann knows that he knows about the talk.

## Example I

Yes, if

1. Ann knows about the talk.
2. Bob knows about the talk.
3. Ann knows that Bob knows about the talk.
4. Bob *does not* know that Ann knows that he knows about the talk.
5. *And nothing else.*

## Social Software: Important Issues

However, finding appropriate representations of beliefs in a multiagent setting has its own problems.

## Brandenburger and Keisler: A Puzzle

Ann believes that Bob assumes\* that  
Ann believes that Bob's assumption is wrong.

Does Ann believe that Bob's assumption is wrong?

\* An assumption (or strongest belief) is a belief that implies all other beliefs.

## Brandenburger and Keisler: Main Result

*No belief model can be complete for a language that contains first-order logic*

**Belief Model:** a set of states for each player, and a relation for each player that specifies when a state of one player considers a state of the other player to be possible.

**Language:** the language used by the players to formulate their beliefs

**Complete:** A belief model is complete for a language if every statement in a player's language which is possible (i.e. true for some states) can be assumed by the player.

## Social Software: Important Issues

Unlike computers, people do not always do what they are told. Thus, purely algorithmic solutions may not always work correctly.

## Example II

Ann and Bob each own a horse and each would like to sell their horse.

Charles is willing to pay \$100 to the owner whose horse can run the **fastest**.

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Ann and Bob each own a horse and each would like to sell their horse.

Charles is willing to pay \$100 to the owner whose horse can run the **fastest**.

In fact, Ann's horse is faster running at 30 MPH, while Bob's horse can only run at 25 MPH.

The obvious procedure that Charles can use to determine who to give the money to is to ask Ann and Bob to race, and give \$100 to the winner.

## Example II

	0	10	20	25
0	0,0	0,100	0,100	0,100
10	100,0	0,0	0,100	0,100
20	100,0	100,0	0,0	0,100
25	100,0	100,0	100,0	0,0
30	100,0	100,0	100,0	100,0

## Example II

	0	10	20	25
0	0,0	0,100	0,100	0,100
10	100,0	0,0	0,100	0,100
20	100,0	100,0	0,0	0,100
25	100,0	100,0	100,0	0,0
30	100,0	100,0	100,0	100,0

On the other hand, if Charles wants to buy the *slower* horse, the above procedure will no longer work.

## Social Software: **Important Issues**

Limited resources may conflict with the assumption that each agent maximizes its individual utility.

## Santa Fe Bar Problem

There is a certain Irish bar in Santa Fe that plays music on Tuesday nights.

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Suppose that

- the bar can comfortably seat 40 people and there are 100 people want to go on Tuesday.
- each agents' preference is *if the bar is not crowded, then the agent would rather go to the bar; and if the bar is crowded, then the agent prefers not to go to the bar.*
- each agent has the same data about the attendance at the bar on previous weeks.

## Santa Fe Bar Problem

There is a certain Irish bar in Santa Fe that plays music on Tuesday nights.

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- each agents' preference is *if the bar is not crowded, then the agent would rather go to the bar; and if the bar is crowded, then the agent prefers not to go to the bar.*
- each agent has the same data about the attendance at the bar on previous weeks.

Is there a theory the agents can follow that will tell them whether or not to go to the bar *this week*?

## Social Software: Important Issues

Group preferences are not necessarily “well-behaved”.

## Condorcet Paradox

Suppose that there are three agents choosing between three alternatives.

$$P_1 \quad a > b > c$$

$$P_2 \quad b > c > a$$

$$P_3 \quad c > a > b$$

Pairwise majority voting produces a non-transitive group preference.

$$\begin{array}{ll} P_1 & a > b > c \\ P_2 & b > c > a \\ P_3 & c > a > b \end{array}$$

- $a > b$ ?
- $b > c$ ?
- $a > c$ ?

- |       |             |
|-------|-------------|
| $P_1$ | $a > b > c$ |
| $P_2$ | $b > c > a$ |
| $P_3$ | $c > a > b$ |
- $a > b?$
  - $b > c?$
  - $a > c?$

$$\begin{array}{ll} P_1 & a > b > c \\ P_2 & b > c > a \\ P_3 & c > a > b \end{array}$$

- $a > b$ ? Yes
- $b > c$ ?
- $a > c$ ?

$$\begin{array}{ll} P_1 & a > b > \textcolor{blue}{c} \\ P_2 & \textcolor{blue}{b} > c > a \\ P_3 & c > a > b \end{array}$$

- $a > b$ ? Yes
- $b > c$ ? Yes
- $a > c$ ?

$$\begin{array}{ll} P_1 & a > b > c \\ P_2 & b > \textcolor{red}{c > a} \\ P_3 & \textcolor{red}{c > a} > b \end{array}$$

- $a > b$ ? Yes
- $b > c$ ? Yes
- $a > c$ ? No

## The Doctrinal Paradox

$P$ : "UvA teachers get a 10% raise"

$Q$ : "The quality of education for all students will increase"

$P \rightarrow Q$ : "If UvA teachers get a 10% raise, then the quality of education for all students will increase"

	$P$	$P \rightarrow Q$	$Q$
Individual 1	True	True	True
Individual 2	True	False	False
Individual 3	False	True	False
Majority	True	True	False

See [personal.lse.ac.uk/LIST/doctrinalparadox.htm](http://personal.lse.ac.uk/LIST/doctrinalparadox.htm)

## Social Software: Important Issues

Agents may have incentives to *misrepresent* their preferences.

## Solomon's Predicament

Suppose two women, 1 and 2, both claim a baby.

Each of them knows who the mother is, but neither can prove her motherhood.

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**Solomon's Solution:** Threaten to cut the baby in two, then give the baby to the women that announces that she prefers the baby to be given to the other women than be cut in two.

*However, the women who is not the real mother has an incentive to misrepresent her preference.*

## Adjusted Winner: Strategizing

Can the agents improve their allocation by misrepresenting their preferences?

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Yes

*However, while honesty may not always be the best policy it is the only safe one, i.e., the only one which will guarantee 50%.*

See *Safe votes, sincere votes, and strategizing* (R. Parikh and EP) for more on the notion of a “safe” strategy.

## Adjusted Winner: Strategizing

Item	Ann	Bob
Matisse	75	25
Picasso	25	75

Ann will get the Matisse and Bob will get the Picasso and each gets 75 of his or her points.

## Adjusted Winner: Strategizing

Suppose Ann knows Bob's preferences, but Bob does not know Ann's.

	Item	Ann	Bob
M	75	25	
P	25	75	

So Ann will get  $M$  plus a portion of  $P$ .

According to Ann's announced allocation, she receives 50 points

According to Ann's actual allocation, she receives  
 $75 + 0.33 * 25 = 83.33$  points.

## Adjusted Winner: Strategizing

Suppose *both* players know each other's preferences but neither knows that the other knows their own preference.

	Item	Ann	Bob		Item	Ann	Bob
	$M$	75	25		$M$	26	74
	$P$	25	75		$P$	74	26

Each will get 74 of his or her announced points, but each one is really getting only 25 of his or her *true* points.

## Adjusted Winner: Strategizing

Suppose *both* players know each other's preferences. Moreover, Ann knows that Bob knows her preference and Bob doesn't know that Ann knows.

Item	Ann	Bob
$M$	26	74
$P$	74	26

---

Item	Ann	Bob
$M$	73	74
$P$	27	26

---

*And so on . . .*

## (Tentative) Course Topics

1. Game Theory/Mechanism Design
2. Social Choice Theory
3. (Formal) Theories of Knowledge
4. Towards a Theory of Correctness of Social Procedures:
5. Fair Division Algorithms
6. Voting Theory
7. ...

## More Specifically

- Savage, Aumann & Anscombe representation result
- Extensive Games, Strategic Games, Equilibrium Concepts, Simple Games
- Arrow's Theorem, Gibbard-Satterthwaite Theorem
- Aumann's Theorem: ‘Agreeing to Disagree’ and related results
- Epistemic Logic, Aumann Structures, History-Based Structures, Probabilistic Models
- Group Knowledge: Common Knowledge, Levels of Knowledge, Coordination Problems
- Game Logic, Coalitional Logic, ATL
- Fair Division Algorithms, Voting Procedures
- ...

## Relevant Courses

- Multiagent Systems (this semester): Ulle Endriss
- Seminar on Dynamic Epistemic Semantics (last semester):  
Johan van Benthem
- Game Theory for Information Sciences (last semester): Peter  
van Embde Boas
- A new course next year: Krzysztof Apt

The sciences do not try to explain, they hardly even try to interpret, they mainly make models. By a model is meant a mathematical construct which, with the addition of certain verbal interpretations, describes observed phenomena. The justification of such a mathematical construct is solely and precisely that it is expected to work. — Johann Von Neumann