Lecture 12: Topics in Voting Theory

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Caput Logic, Language and Information: Social Software

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Introduction

- Introduction to the mathematics of voting procedures
- Two new voting procedures
- Can polls change an election outcome?

Moulin. *Axioms of Cooperative Decision Making (Chapter 9)*. Cambridge University Press (1988).

Brams and Fishburn. *Voting Procedures*. Handbook of Social Choice and Welfare (2002).

For a geometric perspective see

D. Saari. *Basic Geometry of Voting*. Springer (1995).

Voting Problem

Given a (finite!) set X of **candidates**

and a (finite!) set A of **voters**

each of whom have a **preference** over X

Devise a method F which aggregates the individual preferences to produce a collective decision (typically a subset of X)

Voting Procedures

- Type of vote, or **ballot**, that is recognized as admissible by the procedure: let $\mathcal{B}(X)$ be the set of admissible ballots for a set X of candidates
- A method to **count** a vector of ballots (one ballot for each voter) and select a winner (or winners)

Voting Procedures

- Type of vote, or **ballot**, that is recognized as admissible by the procedure: let $\mathcal{B}(X)$ be the set of admissible ballots for a set X of candidates
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Formally, A voting procedure for a set A of agents (with $|A| = n$) and a set X of candidates is a pair

$$(\mathcal{B}(X), \mathsf{Ag})$$

- $\mathcal{B}(X)$ is a set of ballots; and
- $\mathsf{Ag} : \mathcal{B}(X)^n \rightarrow 2^X$ (typically we are interested in the case where $|\mathsf{Ag}(\vec{b})| = 1$).

Examples

Plurality (Simple Majority)

- $\mathcal{B}(X) = X$
- Given $\vec{b} \in X^n$ and $x \in X$, let $\#_x(\vec{b}) = \sum_{\{i \mid b_i = x\}} 1$

$$\mathsf{Ag}(\vec{b}) = \{x \mid \#_x(\vec{b}) \text{ is maximal}\}$$

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Approval Voting

- $\mathcal{B}(X) = 2^X$
- $\mathsf{Ag}(\vec{b}) = \{x \mid \#_x(\vec{b}) \text{ is maximal}\}$

Comparing Voting Procedures

- Does a procedure truly *reflect* the will of the population?

- Some Properties:
  - Pareto Optimality: If a candidate x is unanimously preferred to candidate b, then b should not be elected

  - Anonymity: The name of the voters does not matter (if two voters change votes, then the outcome is unaffected)

  - Neutrality: The name of the candidates does not matter (if two candidate are exchanged in every ranking of the candidates, then the outcome changes accordingly)

  - Monotonicity: Moving up in the rankings is always better

Reflecting the will of the population

# voters	3	5	7	6
	a	a	b	c
	b	c	d	b
	c	b	c	d
	d	d	a	a

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Plurality Winner: a with 8 votes

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Plurality Winner: a with 8 votes

However, a strong majority 13 rank a last!

Reflecting the will of the population

# voters	3	5	7	6
	a	a	b	c
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Condorcet Winner: c beats every other candidate in a pairwise election.

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$$c \quad vs. \quad b$$

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$$c \;\; vs. \;\; d$$

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	a	a	b	c
	b	c	d	b
	c	b	c	d
	d	d	a	a

Condorcet Winner: a candidate a such that for all $b \neq a$, more voters prefer a over b than b over a

Condorcet Consistent Voting Rule: A voting rule that always elects the Condorcet winner (if one exists)

Reflecting the will of the population

# voters	3	5	7	6
	a	a	b	c
	b	c	d	b
	c	b	c	d
	d	d	a	a

Borda: Take into acount the entire ordering.

Reflecting the will of the population

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$$b \ \ vs. \ \ c$$

7 rank b first
6 rank c first
8 rank a first

Reflecting the will of the population

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	d	d	a	a

Borda: Take into acount the entire ordering.

$$b \ \ vs. \ \ c$$

16 rank b first or second

11 rank c first or second

Reflecting the will of the population

# voters	3	5	7	6
	a	a	b	c
	b	c	d	b
	c	b	c	d
	d	d	a	a

Borda: Take into acount the entire ordering.

$$b \ \ vs. \ \ c$$

all voters rank b first, second or third
all voters rank c first, second or third

Reflecting the will of the population

# voters	3	5	7	6
	a	a	b	c
	b	c	d	b
	c	b	c	d
	d	d	a	a

Borda: Take into acount the entire ordering.

$$b \ \ vs. \ \ c$$

Thus, b better reflects the preferences of the population.

Borda Count

# voters	3	5	7	6
	a	a	b	c
	b	c	d	b
	c	b	c	d
	d	d	a	a

Borda Count

# voters	3	5	7	6
3	a	a	b	c
2	b	c	d	b
1	c	b	c	d
0	d	d	a	a

Borda: Take into acount the entire ordering.

- $BC(a) = 3 \times 3 + 3 \times 5 + 0 \times 7 + 0 \times 6 = 24$
- $BC(b) = 2 \times 3 + 1 \times 5 + 3 \times 7 + 2 \times 6 = 44$
- $BC(c) = 1 \times 3 + 2 \times 5 + 1 \times 7 + 3 \times 6 = 29$
- $BC(d) = 0 \times 3 + 0 \times 5 + 2 \times 7 + 1 \times 6 = 20$

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Scoring Rule

Fix a nondecreasing sequence of real numbers

$$s_1 \leq s_1 \leq \cdots \leq s_{m-1}$$

with $s_0 < s_{m-1}$

Voters rank the candidates, thus giving s_0 points to the one ranked last, s_1 to the one ranked next to last, and so on. A candidate with the maximal totla score is elected.

Theorem (Fishburn) There are profiels where the Condorcet winner is never elected by **any** scoring method.

Failure of Monotonicity

Plurality with runoff: In the first round each voter casts a vote for one candidate. If a conadidate wins a strict majority of voites, he is elected. Otherwise, a runoff by majority voting is called between the two candidates that received the most votes in the first round.

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# voters	6	5	4	2
	a	c	b	b
	b	a	c	a
	c	b	a	c

# voters	6	5	4	2
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The profiles are monotonic.

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a wins with the first profile, but c wins with the second.

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# voters	6	5	4	2
	a	x	b	b
	b	a	x	a
	x	b	a	x

# voters	6	5	4	2
	a	c	x	a
	x	a	c	x
	c	x	a	c

a wins with the first profile, but c wins with the second.

No-show Paradox

Totals	Rankings	H over W	W over H
417	B H W	417	0
82	B W H	0	82
143	H B W	143	0
357	H W B	357	0
285	W B H	0	285
324	W H B	0	324
1608		**917**	**691**

Fishburn and Brams. *Paradoxes of Preferential Voting*. Mathematics Magazine (1983).

No-show Paradox

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B: $417 + 82 = 499$

H: $143 + 357 = 500$

W: $285 + 324 = 609$

No-show Paradox

Totals	Rankings	H over W	W over H
417	X H W	417	0
82	X W H	0	82
143	H X W	143	0
357	H W X	357	0
285	W X H	0	285
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H Wins

No-show Paradox

Totals	Rankings	H over W	W over H
419	B H W	417	0
82	B W H	0	82
143	H B W	143	0
357	H W B	357	0
285	W B H	0	285
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Suppose two more people show up with the ranking B H W

No-show Paradox

Totals	Rankings	H over W	W over H
419	B H W	417	0
82	B W H	0	82
143	H B W	143	0
357	H W B	357	0
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285	W B X	0	285
324	W X B	0	324
1610		**644**	**966**

B: $419 + 82 = 501$

H: $143 + 357 = 500$

W: $285 + 324 = 609$

No-show Paradox

Totals	Rankings	B over W	W over B
419	B X W	419	0
82	B W X	82	82
143	X B W	143	0
357	X W B	0	357
285	W B X	0	285
324	W X B	0	324
1610		**644**	**966**

W Wins!

Multiple Districts

Totals	Rankings	East	West
417	B H W	160	257
82	B W H	0	82
143	H B W	143	0
357	H W B	0	357
285	W B H	0	285
324	W H B	285	39
1608		**588**	**1020**

Multiple Districts

Totals	Rankings	East	West
417	B H W	160	257
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143	H B W	143	0
357	H W B	0	357
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324	W H B	285	39
1608		**588**	**1020**

B would win both districts!

Young's Theorem

Reinforcement: If two disjoint groups of voters N_1 and N_2 face the saem set of candidates and N_i selects B_i. If $B_1 \cap B_2 \neq \emptyset$, then $N_1 \cup N_2$ should select $B_1 \cap B_2$.

Continuity Suppose N_1 elects candidate a and a disjoint group N_2 elects $b \neq a$. Then there is a m such that $(nN_1) \cup N_2$ chooses a.

Theorem (Young) A voting correspondence is a scoring method iff it satisfies anonymity, neutrality, reinforcement and continuity.

Young. *Social Choice Scoring Functions.* SIAM Journal of Applied Mathematics (1975).

Approval Voting

Theorem (Fishburn) A voting correspondence is approval voting iff it satisfies anonymity, neutrality, reinforcement and

> If a profile consists of exactly two ballots (sets of candidates) A and B with $A \cap B = \emptyset$, then the procedure selects $A \cup B$.

Fishburn. *Axioms for Approval Voting: Direct Proof.* Journal of Economic Theory (1978).

Strategic Voting

Theorem (Gibbard (1973) and Satterthwaite (1975)) If there are at lest three candidates, a voting rule is strategyproof iff it is dictatorial.

We will not discuss these issues today. See

A. Taylor. *Social Choice and the Mathematics of Strategizing.* 2005.

Digression: Fall-Back Bargaining

Voter Type	d=0	d=1	d=2	d=3	Sum	Max
(1, 1, 1)	0	9	12	13	18	3
(1, 1, 0)	3	5	12	13	19	3
(1, 0, 1)	3	5	12	13	19	3
(0, 1, 1)	3	5	12	13	19	3
(1, 0, 0)	1	8	10	13	20	3
(0, 1, 0)	1	8	10	13	20	3
(0, 0, 1)	1	8	10	13	20	3
(0, 0, 0)	1	4	13	13	21	2
Total	**13**	**52**	**91**	**104**		

- FB (Minimax): (0, 0, 0), MV (Minisum): (1, 1, 1)

Combining Approval and Preference

Under Approval Voting (AV), voters are asked which candidates the voter *approves*

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What about asking for **both** pieces of information?

Combining Approval and Preference

Under Approval Voting (AV), voters are asked which candidates the voter *approves*

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What about asking for **both** pieces of information?

Brams and Sanver . *Voting Systems that Combine Approval and Preference.* available at the author's website.

Assumptions

Assume each voter has a (linear) preference over the candidates.

Each voter is asked to rank the candidates from most preferred to least preferred (ties are not allowed).

Voters are then asked to specify which candidates are acceptable.

Consistency Assumption Given two candidates a and b, if a is approved and b is disapproved then a is ranked higher than b.

For example, we denote this approval ranking for a set $\{a, b, c, d\}$ of candidates as follows

$$a\ d \mid c\ b$$

Preference Approval Voting (PAV)

1. If no candidate, or exactly one candidate, receives a majority of approval votes, the PAV winner is the AV winner.

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   (a) If one of these candidates is preferred by a majority to every other majority approved candidate, then he or she is the PAV winner.

   (b) If there is not one majority-preferred candidate because of a cycle among the majority-approved candidates, then the AV winner among them is the PAV winner.

PAV vs. Condorcet

Rule 1

I. 1 voter: $a\ b \mid c$

II. 1 voter: $b \mid a\ c$

III. 1 voter: $c \mid a\ b$

PAV vs. Condorcet

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I.	1 voter:	$a\ b \mid c$
II.	1 voter:	$b \mid a\ c$
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b is the AV winner.

PAV vs. Condorcet

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I.	1 voter:	$a\ b \mid c$
II.	1 voter:	$b \mid a\ c$
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b is the AV winner.

b is also the PAV winner.

PAV vs. Condorcet

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I.	1 voter:	$a\ b \mid c$
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b is the AV winner.

b is also the PAV winner.

a is the Condorcet winner.

PAV vs. Condorcet

Rule 2(a)

I.	1 voter:	$a\ b\ c \mid d$
II.	1 voter:	$b\ c \mid a\ d$
III.	1 voter:	$d \mid a\ c\ b$

PAV vs. Condorcet

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I.	1 voter:	$a\ b\ c \mid d$
II.	1 voter:	$b\ c \mid a\ d$
III.	1 voter:	$d \mid a\ c\ b$

b is the PAV winner.

a is the Condorcet winner.

PAV vs. Condorcet

Rule 2(b)

I.	1 voter:	$d\ a\ b\ c \mid e$
II.	1 voter:	$d\ b\ c\ a \mid e$
III.	1 voter:	$e \mid d\ c\ a\ b$
IV.	1 voter:	$a\ b\ c \mid d\ e$
V.	1 voter:	$c \mid b\ a\ d\ e$

PAV vs. Condorcet

Rule 2(b)

I.	1 voter:	$d\ a\ b\ c \mid e$
II.	1 voter:	$d\ b\ c\ a \mid e$
III.	1 voter:	$e \mid d\ c\ a\ b$
IV.	1 voter:	$a\ b\ c \mid d\ e$
V.	1 voter:	$c \mid b\ a\ d\ e$

a (3 votes), b (3 votes), and c (4 votes) are all majority approved.

PAV vs. Condorcet

Rule 2(b)

I.	1 voter:	$d\ a\ b\ c \mid e$
II.	1 voter:	$d\ b\ c\ a \mid e$
III.	1 voter:	$e \mid d\ c\ a\ b$
IV.	1 voter:	$a\ b\ c \mid d\ e$
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a (3 votes), b (3 votes), and c (4 votes) are all majority approved.

c is the PAV winner

PAV vs. Condorcet

Rule 2(b)

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III.	1 voter:	$e \mid d\ c\ a\ b$
IV.	1 voter:	$a\ b\ c \mid d\ e$
V.	1 voter:	$c \mid b\ a\ d\ e$

a (3 votes), b (3 votes), and c (4 votes) are all majority approved.

c is the PAV winner.

d is the Condorcet winner.

Example

I.	3 voters:	$a\ b\ c \mid d$
II.	3 voters:	$d\ a\ c \mid b$
III.	2 voters:	$b\ d\ c \mid a$

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I.	3 voters:	$a\ b\ c \mid d$
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c is approved by all 8 voters.

Example

I.	3 voters:	$a\ b\ c \mid d$
II.	3 voters:	$d\ a\ c \mid b$
III.	2 voters:	$b\ d\ c \mid a$

c is approved by all 8 voters.

There is a top cycle $a > b > d > a$ which are all preferred by majorities to c (the AV winner).

a is the PAV winner

Example

I.	3 voters:	$a\ b\ c \mid d$
II.	3 voters:	$d\ a\ c \mid b$
III.	2 voters:	$b\ d\ c \mid a$

a is the PAV winner.

c is the AV winner.

d is the STV winner.

Example

I.	2 voters:	$a\ c\ b \mid d$
II.	2 voters:	$a\ c\ d \mid b$
III.	3 voters:	$b\ c\ d \mid a$

Example

I.	2 voters:	$a\ c\ b \mid d$
II.	2 voters:	$a\ c\ d \mid b$
III.	3 voters:	$b\ c\ d \mid a$

c is approve by all 7 voters.

a is the least approved candidate.

a is the PAV winner.

$BC(a) = 12$
$BC(c) = 14$

Fallback Voting (FV)

1. Voters indicate all candidates of whom they approve, who may range from no candidate to all candidate. Voters rank only those candidates whom they approve.

2. The highest-ranked candidate of all voters is considered. If a majority agrre on the highest-ranked candidate, this candidate is the FV winner (level 1).

3. If there is no level 1 winner, the next-highest ranked candidate of all voters in considered. If a majority of voters agree on one candidate as either their highest or their next-highest ranked candidate, this candidiate is the FV winner (level 2). If more than one receive majority approval, then the candidate with the largest majority is the FV winner.

4. If no level 2 winner, the voters descend – one level at a time — to lower ranks of *approved* candidates stopping when one or more candidates receives majority approval. If more than one receives majority approval then the candidate with the largest majority is the FV winner. If the descent reaches the bottom and no candidate has won, then the candidate with the most approval is the FV winner.

Example

I.	4 voters:	$a\ b\ c \mid d$
II.	3 voters:	$b\ c \mid a\ d$
III.	2 voters:	$d\ a\ c \mid b$

Example

I.	4 voters:	$a\ b\ c \mid d$
II.	3 voters:	$b\ c \mid a\ d$
III.	2 voters:	$d\ a\ c \mid b$

b is the FV winner.

c is the AV winner.

a is the PAV winner.

PAV and FV

- Neither PAV nor FV may elect the Condorcet winner.

- Both PAV and FV are monotonic (approval-monotonic and rank-monotonic)

- Truth-telling strategies of voters under PAV and FV may not be in equilibrium

Polls in elections

How can polls change the outcome of an election?

Example I

The following example is due to Brams and Fishburn

$$P_A^* = o_1 > o_3 > o_2$$
$$P_B^* = o_2 > o_3 > o_1$$
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Size	Group	I	II
4	A	$\mathbf{o_1}$	$\mathbf{o_1}$
3	B	o_2	o_2
2	C	o_3	$\mathbf{o_1}$

If the current winner is o, then agent i will switch its vote to some candidate o' provided

1. o' is one of the top two candidates as indicated by a poll
2. o' is preferred to the other top candidate

Example I

The following example is due to [BF]

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Example II

$$P_A^* = (o_1, o_4, o_2, o_3)$$
$$P_B^* = (o_2, o_1, o_3, o_4)$$
$$P_C^* = (o_3, o_2, o_4, o_1)$$
$$P_D^* = (o_4, o_1, o_2, o_3)$$
$$P_E^* = (o_3, o_1, o_2, o_4)$$

Size	Group	I	II	III	IV
40	A	$\mathbf{o_1}$	o_1	o_4	$\mathbf{o_1}$
30	B	o_2	$\mathbf{o_2}$	$\mathbf{o_2}$	o_2
15	C	o_3	$\mathbf{o_2}$	$\mathbf{o_2}$	o_2
8	D	o_4	o_4	o_1	o_4
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If the current winner is o, then agent i will switch its vote to some candidate o' provided

1. i prefers o' to o, and
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P_A^* &= (o_1, o_4, o_2, o_3) \\
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Towards a Formal Model

A formal model is proposed in which the notion of a protocol is made formal.

Agents change their current vote based on two pieces of information:

1. The poll information
2. The number of agents in i's "group"

Formal details can be found in Chapter 5 of my thesis.

Conclusions

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- Using logic as a tool to analyze game theoretic and social choice problems.
- There are a wealth of social procedures described in the game theory literature. Attempts to formalize these procedures create a wide range of interesting problems for logicians. It will be very interesting to see whether a logical analysis can help create new procedures or refine old ones.

Conclusions

- Logical Omniscience Problem.

  "Deliberation, to the extent that it is thought of as a rational process of figuring out what one should do given one's priorities and expectations is an activity that is unnecessary for the deductively omniscient."— Robert Stalnaker

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- Empirical Studies

Final paper is due May 29, 2006

Send the paper to `epacuit@science.uva.nl`.