

Dynamics for Inference

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1 Introduction

Consider the following two situations:

(DS) from $P \vee Q$, and $\neg P$, infer Q .

and

(WS) I order beef, Jesse orders fish, Darko orders veg. A new waiter walks in with three dishes. What can he do to know what to assign to whom?

In the first case, at the initial state there are four possibilities; at the second state (updated by $P \vee Q$), there are three; at the third (updated by $\neg P$), there is one; and, finally, at the fourth (infer Q), there is one. So it looks like the inference to Q adds no semantic information. But clearly one would not say that an agent cannot come to learn that Q by employing inferences of the (DS) form. Likewise, in the second case: by asking only two questions, the waiter can then use an inference to learn to whom to give the remaining dish. In both cases, notice, two updates, or observations, and one inference are required. Our question, in what follows, concerns how to systematically model the information obtained by employing deductive inferences, and its relationship to the information obtained through update or observation.

2 Inference Dynamics

Below are described classes of models for explicit and implicit information, and their corresponding logics. The crucial feature of the language, the models, and the logic is that it allows us to focus on the engines that drive both explicit and implicit inferences, in contexts where assumptions and conclusions may or may not be true (the informational context), and where

they are required to be true (the knowledge context). First up, the logic EI, in which an agent's explicit info is given by a set of formulae and rules. Then, \mathbf{EI}_K , where formulae and rules are required to be true.

2.1 The Logic EI

Def 1 (Formulae and Rules) Let \mathbf{P} be a set of atoms. \mathcal{I} is called the internal language, in which rules based on \mathcal{I} are pairs (Γ, γ) with Γ finite, γ a singleton. If $\rho = (\Gamma, \gamma)$ is a rule, then $\Gamma = \text{prem}(\rho)$ and $\gamma = \text{conc}(\rho)$.

Def 2 (EI Models) Let

$$M = \langle W, R, V, Y, Z \rangle$$

where $\langle W, R, V \rangle$ is a Kripke model and

$$Y : W \rightarrow \wp(\mathcal{I})$$

satisfies $\gamma \in Y(w)$ and Rwu , then $\gamma \in Y(u)$ and

$$Z : W \rightarrow \wp(\mathcal{R})$$

satisfies $\rho \in Z(w)$ and Rwu , then $\rho \in Z(u)$.

Thm 1 The logic EI is sound and strongly complete.

- Note that $\mathcal{I}(\gamma \rightarrow \delta) \rightarrow (\mathcal{I}\gamma \rightarrow \mathcal{I}\delta)$ is not valid.

2.2 The Logic \mathbf{EI}_K

Def 3 (\mathbf{EI}_K Models) \mathbf{EI}_K models are subsets of the class of **EI** models s.t.:

- R is an equivalence relation;
- for all w , if $\gamma \in Y(w)$, then $(M, w) \Vdash \gamma$ (truth for formulae);
- for all w , if $\rho \in Z(w)$, then $(M, w) \Vdash TR(\rho)$ (truth for rules).

Thm 2 The logic \mathbf{EI}_K is sound and strongly complete.

- Note that $\mathcal{I}\gamma \rightarrow \Box\gamma$ is a theorem of \mathbf{EI}_K .

Def 4 (Deduction Operation) Let M be as before and σ a rule in \mathcal{R} . Then

$$M_\sigma = \langle W, R, V, Y', Z \rangle$$

is given by:

$$Y'(w) := Y(w) \cup \{\text{conc}(\sigma)\}$$

if $\text{prem}(\sigma) \subseteq Y(w)$ and $\sigma \in Z(w)$; and $Y(w)$ otherwise.

- Note: if σ is a rule and M is a model in \mathbf{EI}_K , then M_σ is in \mathbf{EI}_K .
- $\langle D_\sigma \rangle \phi$ reads: *there is a deductive inference with σ yielding ϕ .*
- $(M, w) \Vdash \langle D_\sigma \rangle \phi$ iff $(M, w) \Vdash \mathcal{I}(\text{prem}(\sigma)) \wedge \mathcal{L}\sigma$ and $(M, w) \Vdash \phi$.*

3 Update Dynamics

Now we turn to van Benthem style update dynamics. Consider the following two types of assertions:

- (a) “Reggie saw a red car pass”;
- (b) “Reggie saw that a red car passed”.

Type (a) seems to be a kind of implicit observation: Reggie may have seen a red car pass, but not explicitly know that it was a red car passing. Type (b) seems, then, to be an explicit observation: when it passed, Reggie saw that it was a red car passing. So, type (a) changes the agent’s range of ponderable states, while type (b) changes what the agent explicitly knows. The concern here is similar to that in §2. But whereas in EI one adds formulae and rules in order to study inference patterns under various conditions, here we add access sets (essentially, formulae) in order to study observational patterns under various conditions. The main question, and its solution through the notion of *realization*, is how to get from type (a) to type (b).

3.1 Implicit/Explicit Observation

Def 5 (Epistemic Access Models) EA models are epistemic models s.t.

$$\mathbf{M} = \langle W, W', R, V \rangle$$

where W' is a pair (w, A) with $w \in W$ and A a set of factual propositions such that every formula in A is true at (w, A) .¹

Note the parallel with Velázquez's setup. The difference, so far, is that EA models do not also have rules.

Def 6 Let $(\mathbf{M}, (w, A))$ be an EA model s.t. the actual world satisfies factual formulae ϕ . Then:

6.1 (implicit observation) $!\phi$ is a function from $(\mathbf{M}, (w, A))$ to $(\mathbf{M} \upharpoonright \phi, (w, A))$, where

$$\mathbf{M} \upharpoonright \phi = (W, \{(w, A) \in W' \mid (\mathbf{M}, (w, A)) \Vdash \phi\}, R, V).$$

6.2 (explicit observation) $+\phi$ is a function from $(\mathbf{M}, (w, A))$ to $(\mathbf{M} \upharpoonright^+ \phi, (w, A))$, where

$$\mathbf{M} \upharpoonright^+ \phi = (W, \{(w, A \cup \{\phi\}) \in W' \mid (\mathbf{M}, (w, A)) \Vdash \phi\}, R, V).$$

3.2 Realization

One might think that the (6.1) and (6.2) aren't fine-grained enough, because they don't tell us how an agent goes from implicitly observing that ϕ to explicitly observing that ϕ . Thus, we add an intermediate stage:

Def 7 (realization) $\#\phi$ is a function from $(\mathbf{M}, (w, A))$ to $(\mathbf{M} \upharpoonright^\# \phi, (w, A))$, where

$$\mathbf{M} \upharpoonright^\# \phi = (W, \{w, A \cup \{\phi\}\} \in W' \mid (\mathbf{M}, (w, A)) \Vdash \Box\phi\}, R, V)$$

First, $!\phi$ restricts the agent's information range to only those states satisfying ϕ . Then, provided the agent has the implicit information that ϕ , $\#\phi$ adds ϕ to the access set, making the previously implicit information into explicit information. Thus, $+\phi$ is definable by implicit observation and realization.

¹ One can also define EA models in terms of functions from worlds to access sets (factual formulae), as in §2: $W' : W \rightarrow \wp(A)$ and every $p \in A$ satisfies EI_K 's truth for formula property.

3.3 The Logic EA

Formulae of the language EI are given by:

$$\phi ::= | p | \neg\phi | \phi \vee \psi | \Box\phi | \mathcal{I}\phi | [!\phi]\psi | [\#\phi]\psi$$

and the semantics for, e.g., $[\#\phi]\psi$:

$$(\mathbf{M}, (w, A)) \Vdash [\#\phi]\psi \text{ iff } (\mathbf{M} \upharpoonright^\# \phi, (w, A)) \Vdash \psi.$$

Thm 3 The logic EA is sound and strongly complete.

4 Conclusion

- In contrast to PAL, note that one only puts factual (as opposed to both factual and epistemic) propositions into access sets;
- Similar to EI_K , $\mathcal{I}\gamma \rightarrow \Box\gamma$ is a theorem of EA.
- If an agent has implicit information at a state, then the agent may realize that information. Although this does not mean that the agent has realized the information, it is unclear to what extent the approach deals with the omniscience problem.
- given the above, in contrast to EI_K , realization is more general than deduction, since in EI_K , a formula may be added to access sets on the condition that it is a conclusion to a rule.
- the restriction to factual propositions excludes, e.g., Moore sentences:

$$\neg\Box p \wedge p.$$

How can we extend the program to deal with these?