Logic and Artificial Intelligence

Lecture 11

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$[!\psi]\varphi$: after finding out that ψ, φ is true

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How did you find out that ψ?

- direct observation of ψ
- public announcement of ψ

Incorporating ψ

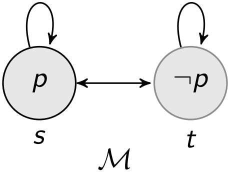
Suppose $\mathcal{M} = \langle W, \{\sim_i\}_{i\in\mathcal{A}}, \{\preceq_i\}_{i\in\mathcal{A}}, V\rangle$ is an epistemic-plausibility model, then $\mathcal{M}^\psi$ is defined as follows:

$\mathcal{M}^\psi = \langle W^\psi, \{\sim_i^\psi\}_{i\in\mathcal{A}}, \{\preceq_i^\psi\}_{i\in\mathcal{A}}, V^\psi\rangle$ with

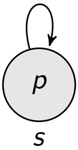
- $W^\psi = W \cap \{w \mid \mathcal{M}, w \models \psi\}$
- For each i, $\sim_i^\psi = \sim_i \cap (W^\psi \times W^\psi)$
- For each i, $\preceq_i^\psi = \preceq_i \cap (W^\psi \times W^\psi)$
- for all $p \in \mathrm{At}$, $V^\psi(p) = V(p) \cap W^\psi$

$$\mathcal{M}, w \models [!\psi]\varphi \text{ iff } \mathcal{M}, w \models \psi \text{ implies } \mathcal{M}^\psi, w \models \varphi$$

Unsuccessful Updates



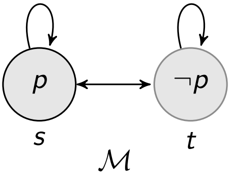
Unsuccessful Updates



$\mathcal{M}^{p \wedge \neg Kp}$

After announcing $p \wedge \neg Kp$

Unsuccessful Updates



$$\mathcal{M}, w \not\models [p \wedge \neg Kp](p \wedge \neg Kp)$$

Unsuccessful Updates

- $[\varphi]\varphi$
- $[\varphi]K_i\varphi$
- $[\varphi]C\varphi$
- **Successful** If $\mathcal{M}, w \models \varphi \wedge \Diamond\varphi$ then $\mathcal{M}^{\varphi}, w \models \varphi$
- **Self-refuting** If $\mathcal{M}, w \models \varphi \wedge \Diamond\varphi$ then $\mathcal{M}^{\varphi}, w \not\models \varphi$

W. Holliday and T. Icard. *Moorean Phenomena in Epistemic Logic*. Proceeding of AiML, 2010.

Finding out vs. conditioning

$[!\psi]B\varphi$ is not equivalent to $B^{\psi}\varphi$:

$[p]B(p \wedge \neg Kp)$ is not satisfiable, but $B^{p}B(p \wedge \neg Kp)$ is satisfiable.

Finding out vs. conditioning

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Probability: Let $cert(\varphi)$ mean $\pi(\varphi) = 1$ and $cert(\varphi \mid \psi)$ mean $\pi(\varphi \mid \psi) = 1$.

Then $\pi(\psi) > 0$ implies $cert(\varphi \mid \psi) \leftrightarrow cert(\psi \to \varphi)$

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$[\pi_i(H) > 0 \wedge T](\pi_i(\pi_i(H) > 0 \wedge T) = 0)$

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$\pi_i(\pi_i(H) > 0 \wedge T \mid \pi_i(H) > 0 \wedge T) = 1$

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  $[\varphi]B\psi \leftrightarrow (\varphi \rightarrow B^{\varphi}[\varphi]\psi)$

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- **Belief**: $[!\psi]B\varphi \not\leftrightarrow (\psi \to B(\psi \to [!\psi]\varphi))$

  $[\varphi]B\psi \leftrightarrow (\varphi \to B^{\varphi}[\varphi]\psi)$
  $[\varphi]B^{\alpha}\psi \leftrightarrow (\varphi \to B^{\varphi \wedge [\varphi]\alpha}[\varphi]\psi)$

The Logic of Public Observation, continued

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The Logic of Public Observation, continued

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- **Make time explicit**: $[!\varphi]CY\varphi$: "After finding out that φ, it is common knowledge that φ *was* true"

But, the agents' *observational* powers may differ...

Dynamic Epistemic Logic

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$$\mathcal{M} \otimes E$$

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Initial epistemic model.

Abstract description of an epistemic event.

Abstract Description of the Event

Ann looks at the card while Bob is looking away.

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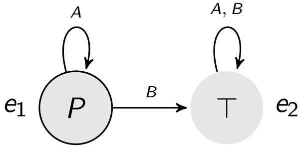

Product Update

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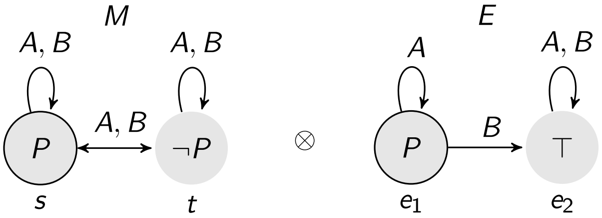

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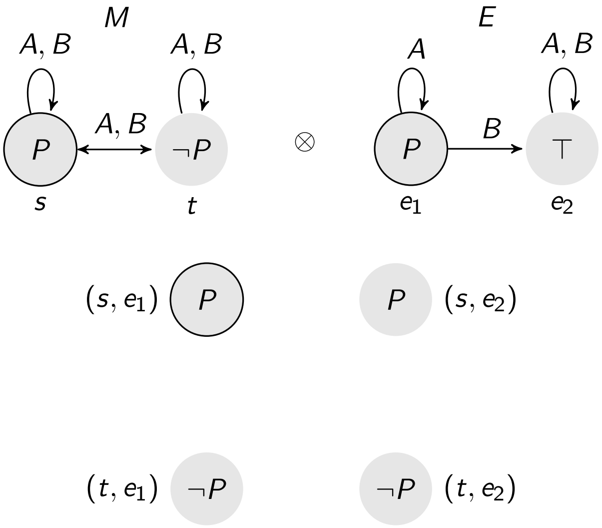

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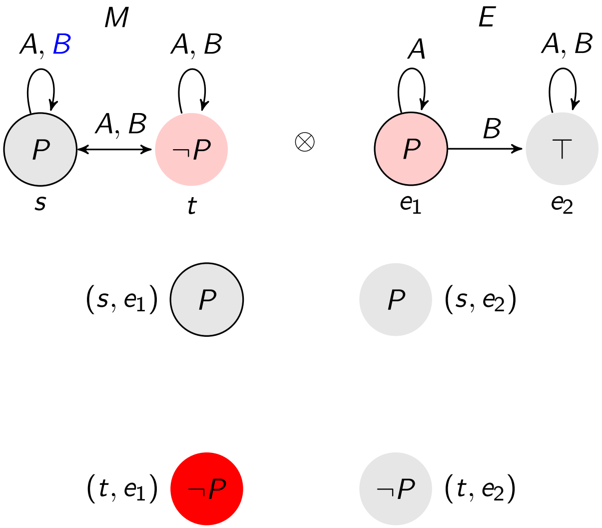

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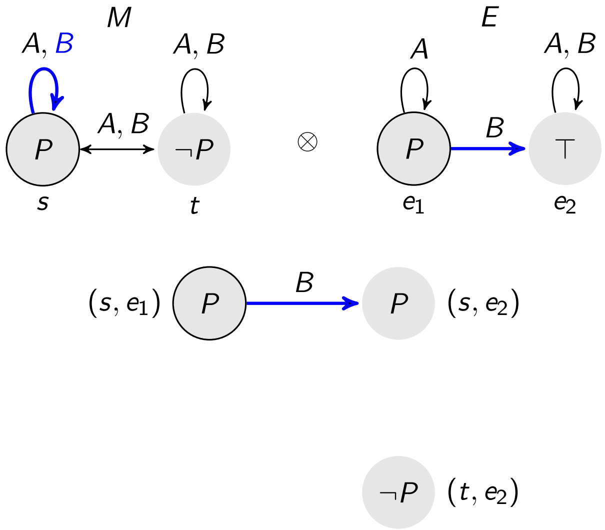

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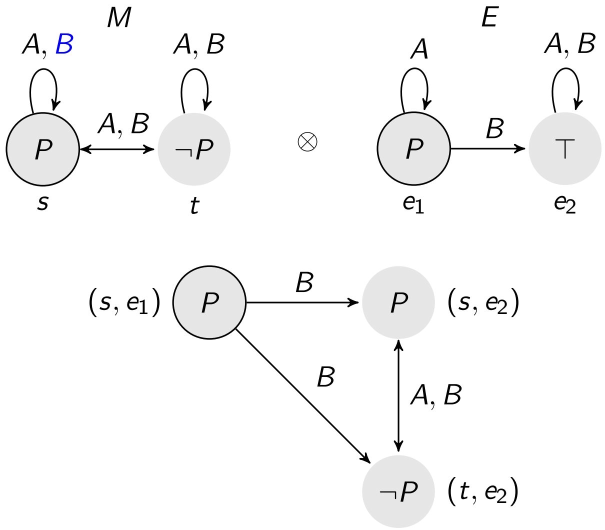

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$\mathcal{M}, w \models [\mathrm{A}, a]\varphi$ iff $\mathcal{M}, w \models Pre(a)$ implies $\mathcal{M} \otimes A, (w, a) \models \varphi$.

Literarture

A. Baltag and L. Moss. *Logics for Epistemic Programs*. 2004.

W. van der Hoek, H. van Ditmarsch and B. Kooi. *Dynamic Epistemic Logic*. 2007.