Logic and Artificial Intelligence

Lecture 20

Eric Pacuit

Currently Visiting the Center for Formal Epistemology, CMU

Center for Logic and Philosophy of Science
Tilburg University

ai.stanford.edu/∼epacuit
e.j.pacuit@uvt.nl

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Actions

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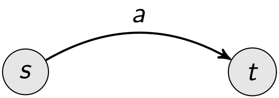

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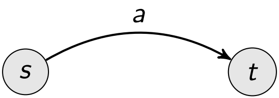
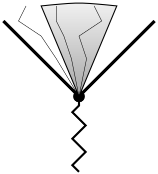
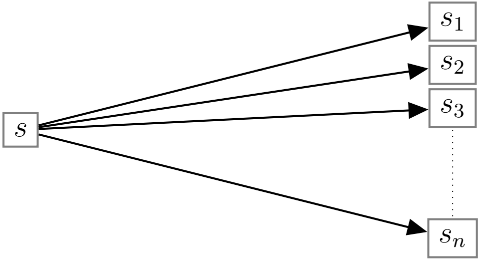


2. Actions *restrict* the set of possible future histories.



J. van Benthem, H. van Ditmarsch, J. van Eijck and J. Jaspers. *Chapter 6: Propositional Dynamic Logic*. Logic in Action Online Course Project, 2011.



s
s_1
s_2
s_3
s_n

Propositional Dynamic Logic

Language: The language of propositional dynamic logic is generated by the following grammar:

$$p \mid \neg\varphi \mid \varphi \wedge \psi \mid [\alpha]\varphi$$

where $p \in \mathsf{At}$ and α is generated by the following grammar:

$$a \mid \alpha \cup \beta \mid \alpha ; \beta \mid \alpha^* \mid \varphi ?$$

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Semantics: $\mathcal{M} = \langle W, \{R_a \mid a \in \mathrm{P}\}, V\rangle$ where for each $a \in \mathrm{P}$, $R_a \subseteq W \times W$ and $V : \mathrm{At} \to \wp(W)$

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$[\alpha]\varphi$ means "after doing α, φ will be true"

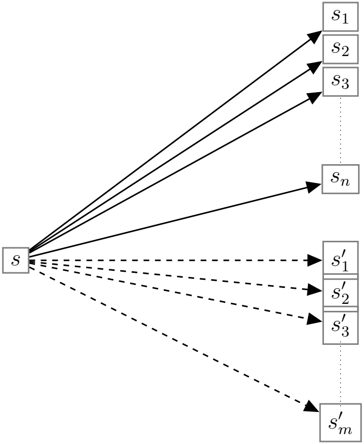
$\langle\alpha\rangle\varphi$ means "after doing α, φ may be true"

$$\mathcal{M}, w \models [\alpha]\varphi \text{ iff for each } v, \text{ if } wR_\alpha v \text{ then } \mathcal{M}, v \models \varphi$$

$$\mathcal{M}, w \models \langle\alpha\rangle\varphi \text{ iff there is a } v \text{ such that } wR_\alpha v \text{ and } \mathcal{M}, v \models \varphi$$

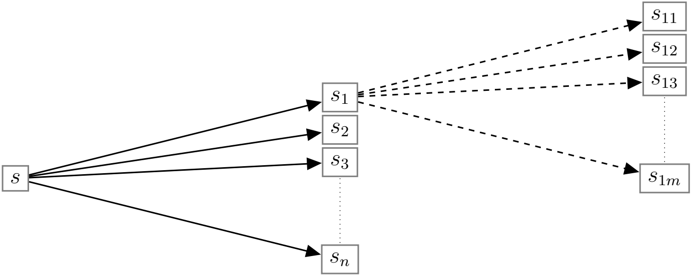
Union

$$R_{\alpha\cup\beta} := R_\alpha \cup R_\beta$$



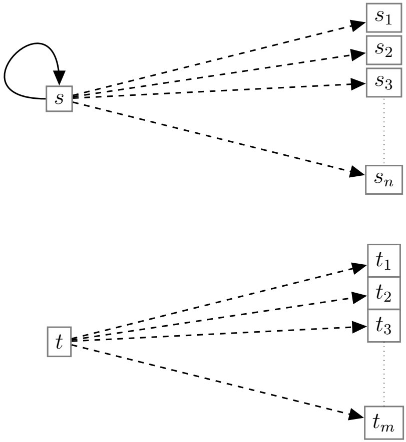
Sequence

$$R_{\alpha;\beta} := R_\alpha \circ R_\beta$$



Test

$$R_{\varphi?} = \{(w,w) \mid \mathcal{M}, w \models \varphi\}$$



Iteration

$$R_{\alpha^*} := \cup_{n \geq 0} R_\alpha^n$$

Propositional Dynamic Logic

1. Axioms of propositional logic
2. $[\alpha](\varphi \to \psi) \to ([\alpha]\varphi \to [\alpha]\psi)$
3. $[\alpha \cup \beta]\varphi \leftrightarrow [\alpha]\varphi \wedge [\beta]\varphi$
4. $[\alpha; \beta]\varphi \leftrightarrow [\alpha][\beta]\varphi$
5. $[\psi?]\varphi \leftrightarrow (\psi \to \varphi)$
6. $\varphi \wedge [\alpha][\alpha^*]\varphi \leftrightarrow [\alpha^*]\varphi$
7. $\varphi \wedge [\alpha^*](\varphi \to [\alpha]\varphi) \to [\alpha^*]\varphi$
8. Modus Ponens and Necessitation (for each program α)

Propositional Dynamic Logic

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4. $[\alpha;\beta]\varphi \leftrightarrow [\alpha][\beta]\varphi$
5. $[\psi?]\varphi \leftrightarrow (\psi \to \varphi)$
6. $\varphi \wedge [\alpha][\alpha^*]\varphi \leftrightarrow [\alpha^*]\varphi$ (Fixed-Point Axiom)
7. $\varphi \wedge [\alpha^*](\varphi \to [\alpha]\varphi) \to [\alpha^*]\varphi$ (Induction Axiom)
8. Modus Ponens and Necessitation (for each program α)

Actions and Ability

An early approach to interpret PDL as logic of actions was put forward by Krister Segerberg.

Segerberg adds an “agency” program to the PDL language δA where A is a formula.

K. Segerberg. *Bringing it about*. JPL, 1989.

Actions and Agency

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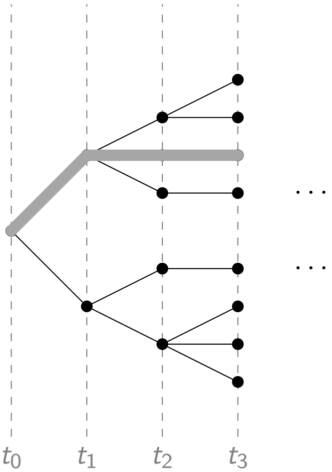
The axioms:

1. $[\delta A]A$
2. $[\delta A]B \rightarrow ([\delta B]C \rightarrow [\delta A]C)$

Actions and Agency in Branching Time

Alternative accounts of agency do not include explicit description of the actions:



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- A **history** is a maximal branch in the above tree.
- Formulas are interpreted at history moment pairs.
- At each moment there is a choice available to the agent (partition of the histories through that moment)
- The key modality is $[i\ stit]\varphi$ which is intended to mean that the agent i can "see to it that φ is true".
  - $[i\ stit]\varphi$ is true at a history moment pair provided the agent can choose a (set of) branch(es) such that every future history-moment pair satisfies φ

STIT

We use the modality '$\Diamond$' to mean historic possibility.

$\Diamond[i\ \textit{stit}]\varphi$: "the agent has the ability to bring about φ".

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- $Choice : \mathcal{A} \times T \to \wp(\wp(H))$ is a function mapping each agent to a partition of H_t
  - $Choice_i^t \neq \emptyset$
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  - For all t and mappings $s_t : \mathcal{A} \to \wp(H_t)$ such that $s_t(i) \in Choice_i^t$, we have $\bigcap_{i \in \mathcal{A}} s_t(i) \neq \emptyset$

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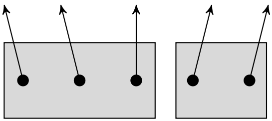

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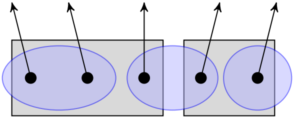

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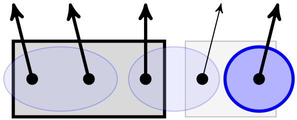

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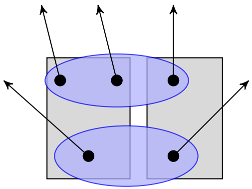

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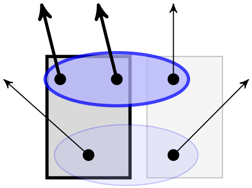

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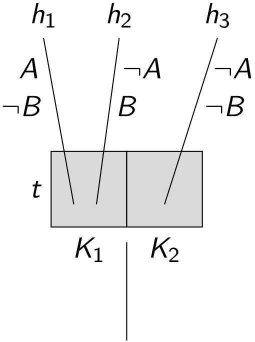
- $\mathcal{M}, t/h \models [i \; stit]\varphi$ iff $\mathcal{M}, t/h' \models \varphi$ for all $h' \in Choice_i^t(h)$
- $\mathcal{M}, t/h \models [i \; dstit]\varphi$ iff $\mathcal{M}, t/h' \models \varphi$ for all $h' \in Choice_i^t(h)$ and there is a $h'' \in H_t$ such that $\mathcal{M}, t/h \models \neg\varphi$

STIT: Example

The following are false: $A \to \Diamond[stit]A$ and $\Diamond[stit](A \vee B) \to \Diamond[stit]A \vee \Diamond[stit]B$.



J. Horty. *Agency and Deontic Logic*. 2001.

STIT: Axiomatics

- **S5** for $\Box$: $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$, $\Box\varphi \to \varphi$, $\Box\varphi \to \Box\Box\varphi$, $\neg\Box\varphi \to \Box\neg\Box\varphi$

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- Modus Ponens and Necessitation for $\Box$

M. Xu. *Axioms for deliberative STIT*. Journal of Philosophical Logic, Volume 27, pp. 505 - 552, 1998.

P. Balbiani, A. Herzig and N. Troquard. *Alternative axiomatics and complexity of deliberative STIT theories*. Journal of Philosophical Logic, 37:4, pp. 387 - 406, 2008.

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- $\Diamond[i\ stit]\varphi$: the agent has the ability to bring about φ