Logic and Artificial Intelligence

Lecture 5

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Knowledge, belief and (un)awareness

Adding Beliefs

Epistemic Models: $\mathcal{M} = \langle W, \{\sim_i\}_{i \in \mathcal{A}}, V \rangle$

Truth: $\mathcal{M}, w \models \varphi$ is defined as follows:

- $\mathcal{M}, w \models p$ iff $w \in V(p)$ (with $p \in \mathsf{At}$)
- $\mathcal{M}, w \models \neg\varphi$ if $\mathcal{M}, w \not\models \varphi$
- $\mathcal{M}, w \models \varphi \wedge \psi$ if $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$
- $\mathcal{M}, w \models K_i\varphi$ if for each $v \in W$, if $w \sim_i v$, then $\mathcal{M}, v \models \varphi$

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Plausibility Relation: $\preceq_i \subseteq W \times W$. $w \preceq_i v$ means

"v is at least as plausibile as w."

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$$Min_{\preceq_i}(X) = \{v \in W \mid v \preceq_i w \text{ for all } w \in X\}$$

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Assumptions:

1. *plausibility implies possibility*: if $w \preceq_i v$ then $w \sim_i v$.
2. *locally-connected*: if $w \sim_i v$ then either $w \preceq_i v$ or $v \preceq_i w$.

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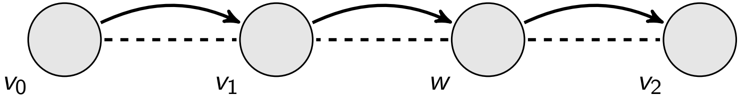
- $\mathcal{M}, w \models K_i\varphi$ if for each $v \in W$, if $w \sim_i v$, then $\mathcal{M}, v \models \varphi$
- $\mathcal{M}, w \models B_i\varphi$ if for each $v \in Min_{\preceq_i}([w]_i)$, $\mathcal{M}, v \models \varphi$
  $[w]_i = \{v \mid w \sim_i v\}$ is the agent's **information cell**.

Grades of Doxastic Strength



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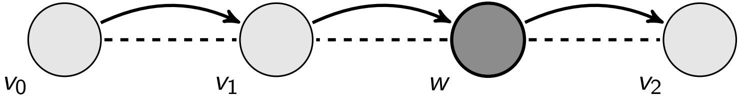

Suppose that w is the current state.

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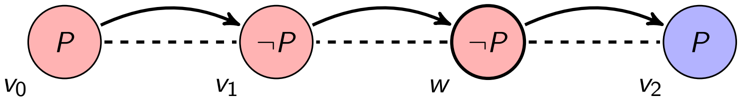

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- **Belief** (BP)

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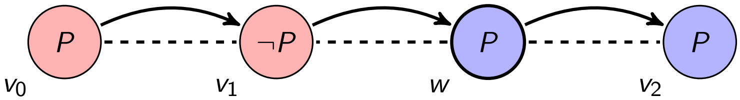

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- **Belief** (BP)
- **Robust Belief** ($\Box P$)

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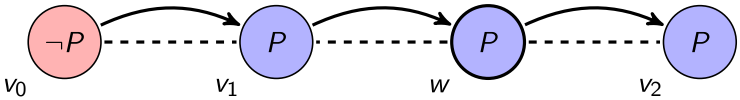

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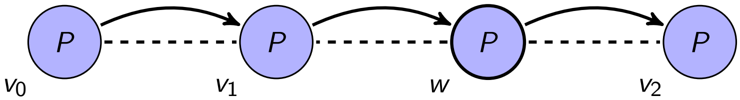

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- **Belief** (BP)
- **Robust Belief** ($\Box P$)
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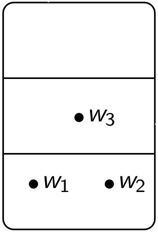
- **Knowledge** (KP)

Conditional Beliefs

- $w_1 \sim w_2 \sim w_3$



Conditional Beliefs

- $w_1 \sim w_2 \sim w_3$
- $w_1 \preceq w_2$ and $w_2 \preceq w_1$ (w_1 and w_2 are equi-plausbile)
- $w_1 \prec w_3$ ($w_1 \preceq w_3$ and $w_3 \not\preceq w_1$)
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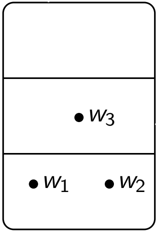

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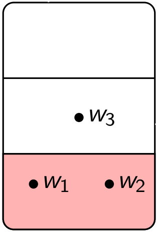
- $\{w_1, w_2\} \subseteq Min_{\preceq}([w_i])$



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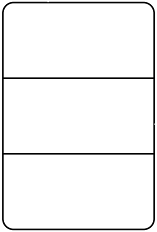
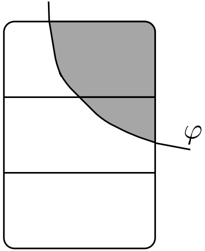


$B_i^{\varphi}\psi$: Agent i believes ψ, *given that φ is true*.

Conditional Beliefs



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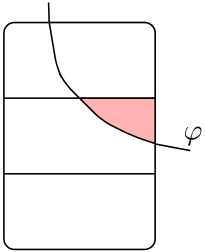

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Core Logical Principles:

1. $B^{\varphi}\varphi$
2. $B^{\varphi}\psi \rightarrow B^{\varphi}(\psi \vee \chi)$
3. $(B^{\varphi}\psi_1 \wedge B^{\varphi}\psi_2) \rightarrow B^{\varphi}(\psi_1 \wedge \psi_2)$
4. $(B^{\varphi_1}\psi \wedge B^{\varphi_2}\psi) \rightarrow B^{\varphi_1 \vee \varphi_2}\psi$
5. $(B^{\varphi}\psi \wedge B^{\psi}\varphi) \rightarrow (B^{\varphi}\chi \leftrightarrow B^{\psi}\chi)$

J. Burgess. *Quick completeness proofs for some logics of conditionals*. *Notre Dame Journal of Formal Logic 22*, 76 – 84, 1981.

Types of Beliefs: Logical Characterizations

- $\mathcal{M}, w \models K_i\varphi$ iff $\mathcal{M}, w \models B_i^{\psi}\varphi$ for all ψ
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- $\mathcal{M}, w \models B_i^s\varphi$ iff $\mathcal{M}, w \models B_i\varphi$ and $\mathcal{M}, w \models B_i^{\psi}\varphi$ for all ψ with $\mathcal{M}, w \models \neg K_i(\psi \to \neg\varphi)$.
  i strongly believes φ iff i believes φ and continues to believe φ given any evidence (truthful or not) that is not known to contradict φ.

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- The agent has not yet entertained possibilities relevant to the truth of φ (the agent is unaware of φ).

Can we model unawareness in state-space models?

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E. Dekel, B. Lipman and A. Rustichini. *Standard State-Space Models Preclude Unawareness*. Econometrica, 55:1, pp. 159 - 173 (1998).

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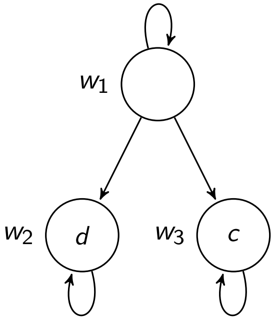
Modeling Watson's Unawareness



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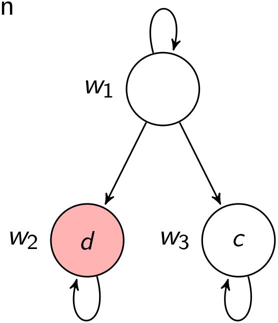
- Let $E = \{w_2\}$ be the event that there was an intruder.



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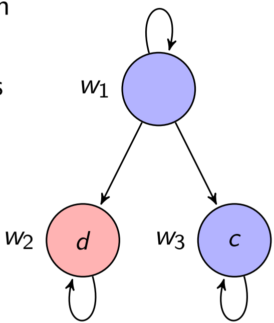

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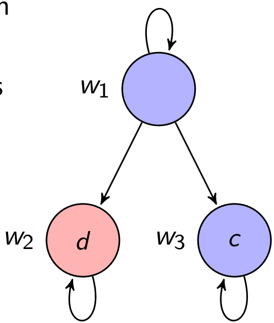

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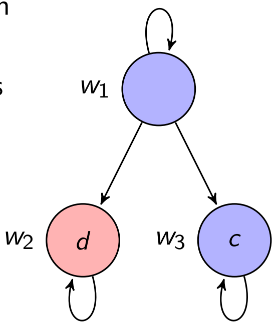
- $-K(E) \cap -K(-K(E)) = \{w_1\}$ and, in fact, $\bigcap_{i=1}^{\infty}(-K)^i(E) = \{w_1\}$



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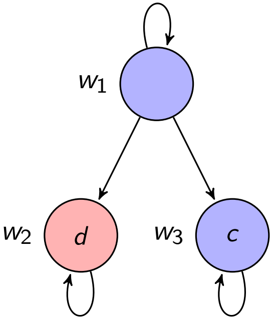

Let $U(F) = \bigcap_{i=1}^{\infty}(-K)^i(F)$. Then,

- $U(\emptyset) = U(W) = U(\{w_1\}) = U(\{w_2, w_3\}) = \emptyset$
- $U(E) = U(\{w_3\}) = U(\{w_1, w_3\}) = U(\{w_1, w_2\} = \{w_1\}$

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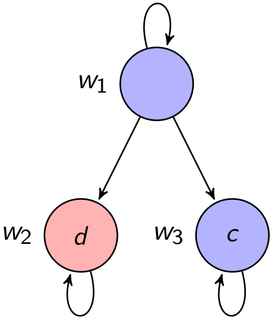

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Then, $U(E) = \{w_1\}$ and $U(U(E)) = U(\{w_1\}) = \emptyset$

Properties of Unawareness

1. $U\varphi \rightarrow (\neg K\varphi \wedge \neg K\neg K\varphi)$

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2. $\neg KU\varphi$
3. $U\varphi \rightarrow UU\varphi$

Theorem. In any logic where U satisfies the above axiom schemes, we have

1. If K satisfies Necessitation (from φ infer $K\varphi$), then for all formulas φ, $\neg U\varphi$ is derivable (the agent is aware of everything); and
2. If K satisfies Monotonicity (from $\varphi \rightarrow \psi$ infer $K\varphi \rightarrow \psi$), then for all φ and ψ, $U\varphi \rightarrow \neg K\psi$ is derivable (if the agent is unaware of something then the agent does not know anything).

B. Schipper. *Online Bibliography on Models of Unawareness*. http://www.econ.ucdavis.edu/faculty/schipper/unaw.htm.

J. Halpern. *Alternative semantics for unawareness*. Games and Economic Behavior, 37, 321-339, 2001.

Epistemic-Probability Models

Adding Probabilities

Epistemic-Probability Model: $\mathcal{M} = \langle W, \{\sim_i\}_{i\in\mathcal{A}}, \{P_i\}_{i\in\mathcal{A}}, V\rangle$ where each $\sim_i$ is an equivalence relation on W is an epistemic model and $P_i : W \rightarrow \Delta(W)$ assigns to each state a probability measure over W, and V is a valuation function.

$\Delta(W) = \{p : W \rightarrow [0,1] \mid p \text{ is a probability measure } \})$

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Write p_i^w for the i's probability measure at state w. We make two natural assumptions:

1. For all $v \in W$, if $p_i^w(v) > 0$ then $p_i^w = p_i^v$; and
2. For all $v \notin [w]_i$, $p_i^w(v) = 0$.

Common Prior

Epistemic-Probabilistic Models: $\mathcal{M} = \langle W, \{\sim_i\}_{i \in \mathcal{A}}, p, V \rangle$

Common Prior: $p : W \to [0, 1]$ is a probability measure (assume W finite)

Truth: $\mathcal{M}, w \models \varphi$ is defined as follows:

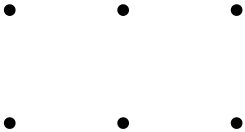
- $\mathcal{M}, w \models p$ iff $w \in V(p)$ (with $p \in \mathrm{At}$)
- $\mathcal{M}, w \models \neg\varphi$ if $\mathcal{M}, w \not\models \varphi$
- $\mathcal{M}, w \models \varphi \wedge \psi$ if $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$
- $\mathcal{M}, w \models K_i\varphi$ if for each $v \in W$, if $w \sim_i v$, then $\mathcal{M}, v \models \varphi$
- $\mathcal{M}, w \models B^r\varphi$ iff $p([\![\varphi]\!] \mid [w]_i) = \frac{p([\![\varphi]\!] \cap [w]_i)}{p([w]_i)} \geq r$

An Example

		Bob L	Bob R
Ann	U	3,3	0,0
Ann	D	0,0	1,1

An Example

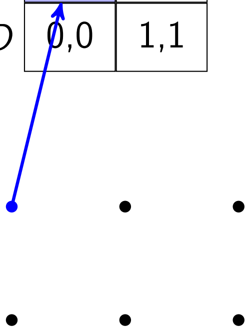
	Bob: *L*	Bob: *R*
Ann: *U*	3,3	0,0
Ann: *D*	0,0	1,1



A set of information states

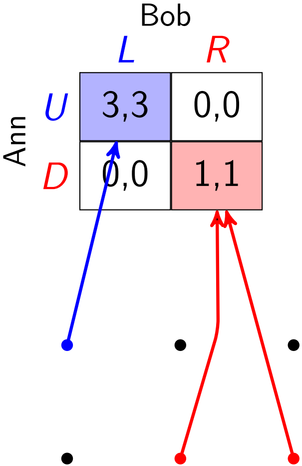
An Example

		Bob: *L*	Bob: *R*
Ann	*U*	3,3	0,0
	D	0,0	1,1



A set of information states

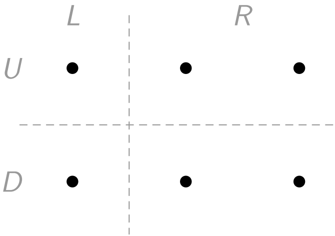
An Example



A set of information states

An Example

Ann \ Bob	L	R
U	3,3	0,0
D	0,0	1,1



A set of information states

An Example

Bob

Ann	L	R
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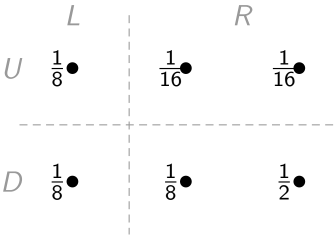
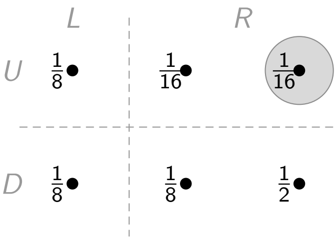


A common prior

An Example

Bob

Ann	L	R
U	3,3	0,0
D	0,0	1,1

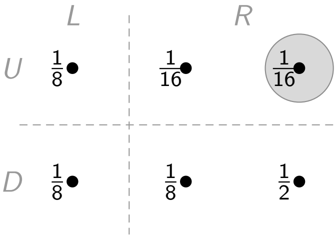


Suppose Ann chooses U and Bob chooses R

An Example

Bob

Ann		L	R
	U	3,3	0,0
	D	0,0	1,1

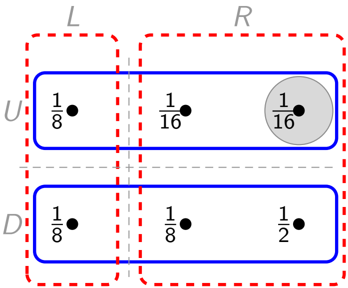


Suppose Ann chooses U and Bob chooses R

Are these choices rational?

An Example

Ann \ Bob	L	R
U	3,3	0,0
D	0,0	1,1



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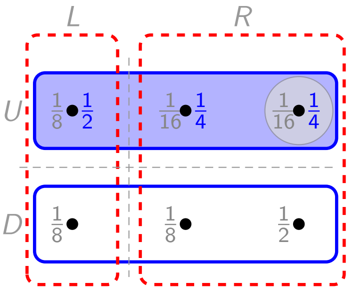
An Example

Bob

Ann	L	R
U	3,3	0,0
D	0,0	1,1



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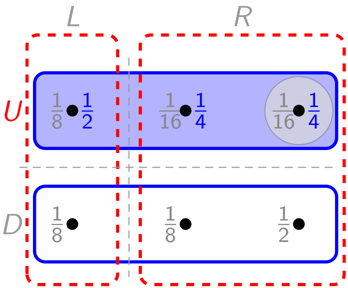


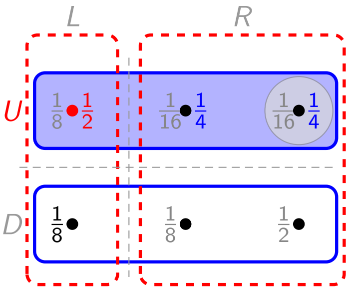
$$3 \cdot P_A(L) + 0 \cdot P_A(R) \geq 0 \cdot P_A(L) + 1 \cdot P_A(R)$$

An Example

Ann \ Bob	L	R
U	3,3	0,0
D	0,0	1,1

Suppose Ann chooses U and Bob chooses R

Are these choices rational?



$$3 \cdot P_A(L) + 0 \cdot P_A(R) \geq 0 \cdot P_A(L) + 1 \cdot P_A(R)$$

An Example

Bob

Ann		L	R
	U	3,3	0,0
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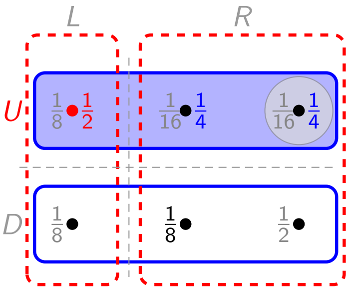


$$3 \cdot \frac{1}{2} + 0 \cdot P_A(R) \geq 0 \cdot \frac{1}{2} + 1 \cdot P_A(R)$$

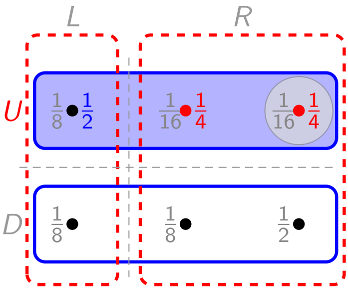
An Example

Bob

Ann	L	R
U	3,3	0,0
D	0,0	1,1

Suppose Ann chooses U and Bob chooses R

Are these choices rational?



$$3 \cdot \frac{1}{2} + 0 \cdot \frac{1}{2} \geq 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2}$$

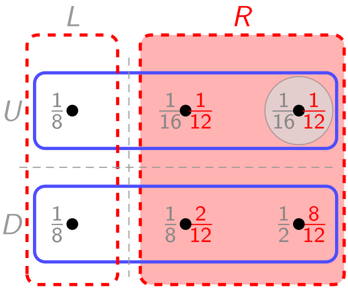
An Example

Bob

Ann	L	R
U	3,3	0,0
D	0,0	1,1

Suppose Ann chooses U and Bob chooses R

Are these choices rational?



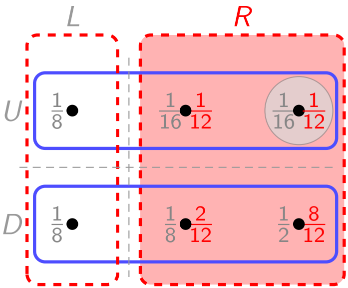
$$0 \cdot P_B(U) + 1 \cdot P_B(D) \geq 3 \cdot P_B(U) + 0 \cdot P_B(D)$$

An Example

		Bob: *L*	Bob: *R*
Ann	*U*	3,3	0,0
	D	0,0	1,1

Suppose Ann chooses U and Bob chooses R

Are these choices rational?

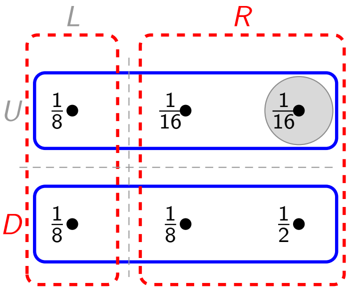


$$0 \cdot \frac{2}{12} + 1 \cdot \frac{10}{12} \geq 3 \cdot \frac{2}{12} + 0 \cdot \frac{10}{12}$$

An Example

Bob

Ann	L	R
U	3,3	0,0
D	0,0	1,1

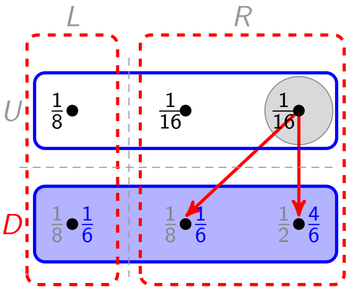


Suppose Ann chooses U and Bob chooses R

Are these choices rational?
Yes.

An Example

		Bob L	Bob R
Ann	U	3,3	0,0
Ann	D	0,0	1,1



Suppose Ann chooses U and Bob chooses R

Are these choices rational?
Yes.

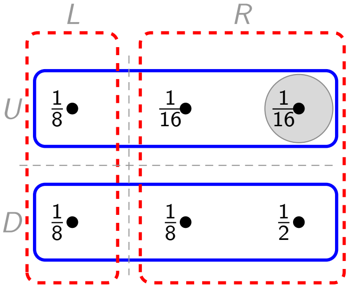
Bob (Ann) *knows* that Ann (Bob) is *rational*

$$0 \cdot \frac{1}{6} + 1 \cdot \frac{5}{6} \geq 3 \cdot \frac{1}{6} + 0 \cdot \frac{5}{6}$$

Two Issues

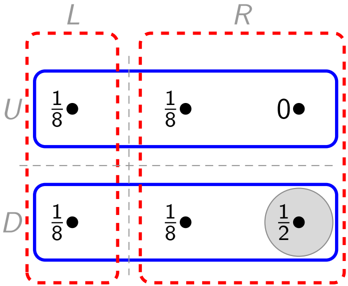
Bob

Ann	L	R
U	3,3	0,0
D	0,0	1,1



Two Issues

		Bob L	R
Ann	U	3,3	0,0
	D	0,0	1,1

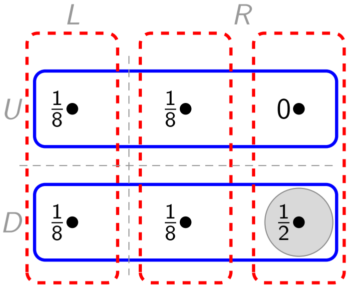


1. Zero probability $\neq$ “impossible”

Two Issues

Bob

		L	R
Ann	U	3,3	0,0
	D	0,0	1,1



1. Zero probability $\neq$ "impossible"
2. Different "types" of players can make the same choice

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Ann \ Bob	L	R
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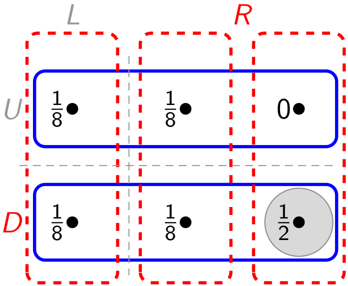

1. Zero probability $\neq$ "impossible"
2. Different "types" of players can make the same choice

- Are Ann and Bob rational?

Two Issues

Ann \ Bob	L	R
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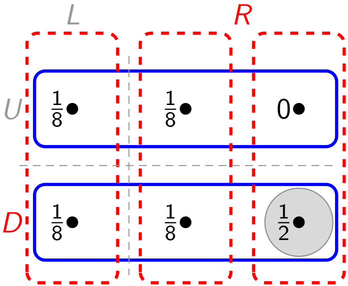
D	0,0	1,1



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2. Different "types" of players can make the same choice

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Two Issues

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		L	R
Ann	U	3,3	0,0
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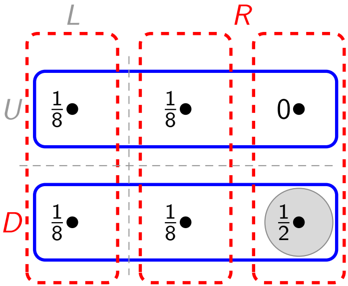

1. Zero probability $\neq$ "impossible"
2. Different "types" of players can make the same choice

- Are Ann and Bob rational? Yes.
- Do they *know* that each other is rational? No.

Two Issues

Ann \ Bob	L	R
U	3,3	0,0
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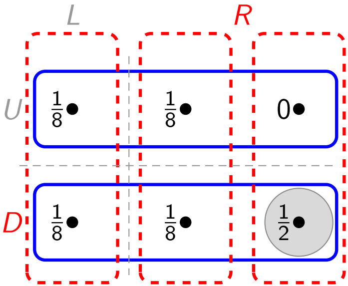

1. Zero probability $\neq$ "impossible"
2. Different "types" of players can make the same choice

- Are Ann and Bob rational? Yes.
- Do they *know* that each other is rational? No.
  ($though\ Pr_{Bob}(Irrat(Ann)) = 0$)

Next: Common Knowledge