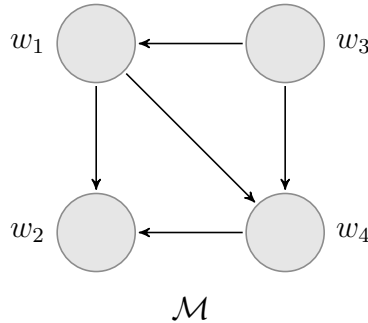


## Problem Set # 1

(all questions have equal weight)

1. Consider the following relational structure (assume that there are no atomic propositions in the language):



For each of the follows sets of states, find a formula that is true at precisely those sets (note that since there are no atomic propositions, the formulas will be construction using  $\perp$  and  $\top$ ):  $\emptyset$ ,  $\{w_1\}$ ,  $\{w_2\}$ ,  $\{w_3\}$ ,  $\{w_4\}$ ,  $\{w_1, w_2, w_3, w_4\}$ .

2. We say a frame  $\langle W, R \rangle$  is **secondary reflexive** if  $R$  has the property  $\forall x \forall y (xRy \rightarrow yRy)$ . Prove that for all frames  $\mathcal{F} = \langle W, R \rangle$ ,  $\mathcal{F} \models \Box(\Box\varphi \rightarrow \varphi)$  iff  $\mathcal{F}$  is secondary reflexive.
3. Which one of the following two implications is valid in multiagent **S5**? Draw a counter-example for the other:

$$L_1 K_2 \varphi \rightarrow L_2 L_1 \varphi$$

$$L_1 K_2 \varphi \rightarrow L_2 K_1 \varphi$$

(recall that  $L_i \varphi$  is defined to be  $\neg K_i \neg \varphi$ )

4. Read the article by Joe Halpern *Should Knowledge Entail Belief?*, Journal of Philosophical Logic (there is a link on the website). Write a short explanation in your own words summarizing Halpern's main point. (That is, explain in 1-2 paragraphs what is Halpern's main message in this article).

**The homework is DUE Monday, September 19.**