Problem Set # 2

1. In this problem we consider a possible definition of common belief, analogous to the definition of common knowledge. Suppose there are two agents and a belief model $\langle W, \{R_1, R_2\}, V\rangle$ where R_1 and R_2 are serial, transitive and Euclidean relations. Let $R_B = (R_1 \cup R_2)^+$, where R^+ is the transitive closure of R (the smallest transitive relation containing R). Define the common belief operator C^B as follows:

$$\mathcal{M}, w \models C^B\varphi \text{ iff for each } v \in W, \text{ if } wR_Bv \text{ then } \mathcal{M}, v \models \varphi$$

   (a) Provide a **KD45** model $\mathcal{M} = \langle W, \{R_1, R_2\}, V\rangle$ and a state $w \in W$ where $\mathcal{M}, w \models B_1(C^Bp)$ but $\mathcal{M}, w \models \neg C^Bp$ (i.e., a state where agent 1 believes that p is commonly believed, but p is, in fact, not commonly believed).

   (b) Provide an example that shows that negative introspection for common belief ($\neg C^B\varphi \rightarrow C^B\neg C^B\varphi$) is not valid

2. We have argued that $K_i\varphi \rightarrow K_j\varphi$ is valid on a frame $\langle W, \{R_i\}_{i\in\mathcal{A}}\rangle$ iff for each $i, j \in \mathcal{A}$, $R_j \subseteq R_i$. Find a property on frames $\langle W, \{R_i\}_{i\in\mathcal{A}}\rangle$ that guarantees that $K_i\varphi \rightarrow K_iK_j\varphi$ is valid.

3. For a Bayesian model with a common prior $\langle W, \{\sim_i\}_{i\in\mathcal{A}}, \pi\rangle$, prove that for each $i \in \mathcal{A}$, $\pi(E \mid B_i^p(E)) \geq p$.

4. Explain why Aumann's original agreeing to disagree theorem (Theorem 7 in the handout for lecture 8) follows from Samet's generalized agreeing to disagree theorem (Theorem 4 in the handout for lecture 8). *Hint: fix and event $E \subseteq W$ and for each agent i, let the decision function $\mathbf{d}_i$ be defined as follows: $\mathbf{d}_i(w) = \pi(E \mid [w]_i)$ (the posterior probability of E for agent i at state w). Prove that $\mathbf{d}$ satisfies the ISTP.*

The homework is DUE Wednesday, October 5.