Problem Set # 5

1. Prove that the following axiom of ceteris paribus logic is valid (see slide 22 of lecture 21 on 11/16):
$$(\alpha \wedge \langle\Gamma\rangle^{\leq}(\alpha \wedge \varphi)) \rightarrow \langle\Gamma \cup \{\alpha\}\rangle^{\leq}\varphi$$

2. Let X, Y be subsets of W and suppose that $\leq$ is a reflexive, connected and transitive order over W. Say $X \leq_{\forall\forall} Y$ provided for all $x \in X$ and for all $y \in Y$, we have $x \leq y$. Assume that $\leq$ is reflexive, transitive and complete, is $\leq_{\forall\forall}$ also reflexive, transitive, and complete? If so, prove it and if not, give a counterexample. Can you think of any other interesting principles that $\leq_{\forall\forall}$ satisfies?

3. Recall the model of knowledge and preference from Lecture 22 (on 11/21): $\mathcal{M} = \langle W, \sim, \preceq, V\rangle$ where $\sim$ is an equivalence relation and $\preceq$ is a reflexive, transitive and total preference relation. Truth is defined as follows:

   - $\mathcal{M}, w \models K\varphi$ iff for all $v \in W$, if $w \sim v$ then $\mathcal{M}, v \models \varphi$
   - $\mathcal{M}, w \models \langle\preceq\rangle\varphi$ iff there is a $v \in W$ with $w \preceq v$ and $\mathcal{M}, v \models \varphi$
   - $\mathcal{M}, w \models A\varphi$ iff for all $v \in W$, $\mathcal{M}, v \models \varphi$
   - $\mathcal{M}, w \models \langle\sim \cap \preceq\rangle\varphi$ iff there is a $v \in W$ such that $w \sim v$ and $w \preceq v$ with $\mathcal{M}, v \models \varphi$

   Given an example to show that $K(\psi \rightarrow \langle\preceq\rangle\varphi)$ and $K(\psi \rightarrow \langle\sim \cap \preceq\rangle\varphi)$ or not equivalent (i.e., find a model and state where one of the formulas is true, but the other is not true). It is easy to see that $A(\psi \rightarrow \langle\preceq\rangle\varphi) \rightarrow K(\psi \rightarrow \langle\preceq\rangle\varphi)$ is valid (this is an instance of the validity $A\varphi \rightarrow K\varphi$, but what is the relationship between $A(\psi \rightarrow \langle\preceq\rangle\varphi)$ and $K(\psi \rightarrow \langle\sim \cap \preceq\rangle\varphi$ (does one imply the other or are the two formulas independent)?

The homework is DUE Monday, December 5th (put you answers in my mailbox).