Rationality

Lecture 12

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Shared cooperative activity

What is a *team*?

Any group?

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- Surely not. But interesting phenomena at this level already.

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Then a group with:

i A certain (hierarchical) structure?

ii Whose members identify with the group (c.f. Gold 2005)?

- Information about who's in and who's out.
- Reasoning and acting as group members.

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iii Team- or group objectives/aims/preferences?

  - Shared by the members?

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iv Shared commitments? (Bratman, 1999, Gilbert 1989, Tuomela, 2007)

  - Shared intentions.
  - Sanctions for lapsing?
  - Shared praise[blame] for success[failure]?

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v Common knowledge (beliefs?) of (i-iv)?

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Note: *None of these are necessary conditions*!

What is a *team*?

Acting as a team (at least) involves:

- Adopting the team's preferences. (Preference transformation).
- Team-reasoning (Agency Transformation).

What is a *team*?

1. Group identification.
   - Information about who's in and who's out.
   - Reasoning as group members.
   - Shared goal.
     - Group preference / utilities.
2. Shared commitments.
   - Shared intentions.
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Intentions: Recap

Motivational attitudes which:

- Are relatively stable.
- Are conduct-controlling, i.e. commit to action.
- Constraint further practical reasoning.

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Commitments and Intentions

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Intentions and Teamwork:

M. Gilbert. *On Social Facts*. Princeton UP, 1989.

J. Searle. *The Construction of Social Reality*. Free Press, 1995.

M. Bratman. *Faces of Intentions*. Cambridge UP, 1999.

R. Tuomela. *The Philosophy of Sociality*. Oxford UP, 2010.

Shared Intentions

A The Intention part:

1. Me:
   1.1 I intend that we J.
   1.2 I intend that we J in accordance with and because of meshing subplans of (1.1) and (2.1).

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3. Additional requirements:
   - 3.1 The intentions in (1) and in (2) are not coerced by the other participant.
   - 3.2 The intentions in (1) and (2) are minimally cooperatively stable.

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B: The epistemic part:

1. It is common knowledge between us that (A).

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        - Group Decision Making
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Main Question

Given a group of people faced with some decision, how should a central authority combine the individual opinions so as to best reflect the "will of the group"?

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Typical Examples:

- Electing government officials
- Department meetings
- Deciding where to go to dinner with friends
- ....

Group Rationality Constraints

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  - Puzzles of Fair Division
- Many proposed group decision methods (voting methods) with very little agreement about how to compare them.

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
↑	B	C	D	B
	C	B	C	D
worst	D	D	A	A

Brams and Fishburn. *Voting Procedures*. Handbook of Social Choice and Welfare (2002).

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A few observations:

- More people rank *A* first than any other candidate

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A few observations:

- More people rank A first than any other candidate
- But, a stronger majority ranks A last

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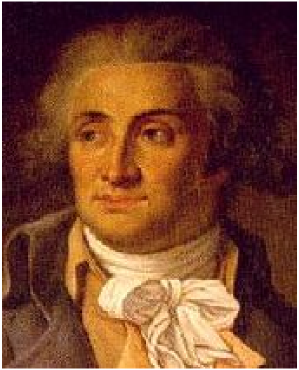

Marquis de Condorcet (1743 - 1794)

vs.

Jean-Charles de Borda (1733 -1799)

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- In pairwise elections, C beats every other candidate (C is the Condorcet winner)

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A few observations:

- More people rank A first than any other candidate
- In pairwise elections, C beats every other candidate (C is the Condorcet winner)
- B and C are the only candidates not ranked last by anyone

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worst	D	D	A	A

A few observations:

- More people rank A first (last) than any other candidate
- In pairwise elections, C beats every other candidate (C is the Condorcet winner)
- Taking into account the *entire* ordering, B has the most "support" (B is the Borda winner)

Which candidate *should* be chosen?

# voters	3	5	7	6
3	A	A	B	C
2	B	C	D	B
1	C	B	C	D
0	D	D	A	A

A few observations:

- More people rank A first (last) than any other candidate
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- B gets $3 \times 2 + 5 \times 1 + 7 \times 3 + 6 \times 2 = 44$ points

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  C gets $3 \times 1 + 5 \times 2 + 7 \times 1 + 6 \times 3 = 38$ points

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Conclusion: *many ways to answer the above question!*

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- **Information required from the voters**: Ordinal vs. cardinal preferences; ranked/non-ranked procedures; does there exist a common "grading language"?
- **Axiomatic results**: Characterize different procedures in terms of abstract normative *properties*.

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- **Neutrality**: The names of the candidates, or options, do not matter (if two candidate are exchanged in every ranking, then the outcome changes accordingly)
- **Monotonicity**: Moving up in the rankings is always better

Fundamental problem(s) of social choice theory

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Condorcet Paradox

Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
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Condorcet Paradox

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Condorcet Paradox

Voter 1	Voter 2	Voter 3
A	C	B
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- Does the group prefer A over B? Yes
- Does the group prefer B over C? Yes
- Does the group prefer A over C? No
  (this conflicts with transitivity)

Doctrinal Paradox

Suppose that three experts *independently* formed opinions about three propositions. For example,

1. p: "Carbon dioxide emissions are above the threshold x"
2. $p \rightarrow q$: "If carbon dioxide emissions are above the threshold x, then there will be global warming"
3. q: "There will be global warming"

Doctrinal Paradox

	p	$p \rightarrow q$	q
Expert 1			
Expert 2			
Expert 3			

Doctrinal Paradox

	p	$p \to q$	q
Expert 1	True	True	
Expert 2			
Expert 3			

Doctrinal Paradox

	p	$p \rightarrow q$	q
Expert 1	True	True	True
Expert 2			
Expert 3			

Doctrinal Paradox

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True		False
Expert 3			

Doctrinal Paradox

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Expert 1	True	True	True
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Doctrinal Paradox

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	p	$p \to q$	q
Expert 1	True	True	True
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Group			

Doctrinal Paradox

	p	$p \to q$	q
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True		

Doctrinal Paradox

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Doctrinal Paradox

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Expert 3	False	True	False
Group	True	True	False

Many Variants!

See
http://personal.lse.ac.uk/LIST/doctrinalparadox.htm
for many generalizations!

Kornhauser and Sager. *Unpacking the court*. Yale Law Journal, 1986.

C. List and P. Pettit. *Aggregating Sets of Judgments: An Impossibility Result*. Economics and Philosophy 18: 89-110, 2002.

F. Dietrich and C. List. *Arrow's theorem in judgment aggregation*. Social Choice and Welfare 29(1): 19-33, 2007.

Example: Characterizing Majority Rule

If there are only **two** options, then majority voting is the "best" procedure:

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If there are only **two** options, then majority voting is the "best" procedure: Choosing the outcome with the most votes (allowing for ties) is the *only* group decision method satisfying the previous properties.

K. May. *A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision*. Econometrica, Vol. 20 (1952).

May's Theorem: Details

Suppose there are only two candidates A and B and n voters (let $N = \{1, \dots, n\}$ be the set of voters).

Then the voters' preferences can be represented by elements of $\{-1, 0, 1\}$ (where 1 means A is preferred to B, -1 means B is preferred to A and 0 means indifference between A and B).

A **social decision method** is a function $F : \{-1, 0, 1\}^n \to \{-1, 0, 1\}$.

May's Theorem: Details

- **Unanimity**: unanimously supported alternatives must be the social outcome.
- **Anonymity**: all voters should be treated equally.
- **Neutrality**: all candidates should be treated equally.
- **Monotonicity**: unidirectional shift in voters' opinions should not harm the alternative toward which this shift occurs

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  $F(v_1, v_2, \ldots, v_n) = F(v_{\pi(1)}, v_{\pi(2)}, \ldots, v_{\pi(n)})$ where π is a permutation of the voters.

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  If $F(v) = 0$ or $F(v) = 1$ and $v \prec v'$, then $F(v') = 1$ (where $v \prec v'$ means for all $i \in N$ $v_i \leq v'_i$ and there is some $i \in N$ with $v_i < v'_i$) then $F(v') = 1$.

May's Theorem: Details

May's Theorem (1952) A social decision method F satisfies unaniminity, neutrality, anonminity and positive responsiveness iff F is majority rule.

Other characterizations

G. Asan and R. Sanver. *Another Characterization of the Majority Rule*. Economics Letters, 75 (3), 409-413, 2002.

E. Maskin. *Majority rule, social welfare functions and game forms*. in *Choice, Welfare and Development*, The Clarendon Press, pgs. 100 - 109, 1995.

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Digression: Infinite Populations

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Plurality, Borda Count, Antiplurality/Veto, and k-approval; Plurality with Runoff; Single Transferable Vote (STV)/Hare; Approval Voting; Condorcet-consistent methods based on the simple majority graph (e.g., Cup Rule/Voting Trees, Copeland, Banks, Slater, Schwartz, and the basic Condorcet rule itself), rules based on the weighted majority graph (e.g., Maximin/Simpson, Kemeny, and Ranked Pairs/Tideman), rules requiring full preference information (e.g., Bucklin, Dodgson, and Young); Majoritarian Judgment; Cumulative Voting; Range Voting

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What happens when there are more than two candidates?

Plurality Vote: Each voter selects one candidate (or none if voters can abstain) and the candidate(s) with the most votes win.

Plurality with Runoff: If there is a candidate with an absolute majority then that candidate wins, otherwise the top two candidates move on to round two. The candidate with the most votes in the second round wins.

What happens when there are more than two candidates?

Approval Voting: Each voter selects a *subset* of the candidates (empty set means the voter abstains) and the candidate(s) with the most votes win.

Borda Count: Each voter provides a linear ordering of the candidates. The candidate(s) with the most total **points** wins, where points are calculated as follows: if there are n candidates, $n-1$ points are given to the highest ranked candidates, $n-2$ to the second highest, etc..

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Failure of monotonicity: plurality with runoff

# voters	6	5	4	2
	A	C	B	B
	B	A	C	A
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Winner: *A*

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# voters	6	5	4	2
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	C	B	A	C

Winner: C

Failure of monotonicity: plurality with runoff

# voters	6	5	4	2
	A	C	B	**B**
	B	A	C	**A**
	C	B	A	**C**

Winner: *A*

# voters	6	5	4	2
	A	C	B	**A**
	B	A	C	**B**
	C	B	A	**C**

Winner: *C*

No-show paradox

Totals	Rankings	H over W	W over H
417	B H W	417	0
82	B W H	0	82
143	H B W	143	0
357	H W B	357	0
285	W B H	0	285
324	W H B	0	324
1608		**917**	**691**

Fishburn and Brams. *Paradoxes of Preferential Voting*. Mathematics Magazine (1983).

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B: $417 + 82 = 499$
H: $143 + 357 = 500$
W: $285 + 324 = 609$

No-show paradox

Totals	Rankings	H over W	W over H
417	X H W	417	0
82	X W H	0	82
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H Wins

No-show paradox

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Suppose two more people show up with the ranking B H W

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1610		**644**	**966**

W Wins!

Multiple Districts

Totals	Rankings	East	West
417	B H W	160	257
82	B W H	0	82
143	H B W	143	0
357	H W B	0	357
285	W B H	0	285
324	W H B	285	39
1608		**588**	**1020**

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B would win both districts!

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Should the Condorcet Winner be Elected?

# Voters	8	5
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	C	B	A

# Voters	13	10	5
	A	B	C
	B	C	A
	C	A	B

More than two candidates

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- Many different procedures (Plurality, Plurality with runoff, Borda Count, Approval)
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Let X be a finite set with *at least three elements*.
Assume each agent has a transitive and complete preference over X (ties are allowed).

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- An **social welfare function** maps an ordering for each agent to a "social ordering" (F is a function from the voters' preferences to a preference, so $F(P_1, \ldots, P_n)$ is an ordering over X.)
- Notation: write $\vec{P}$ for the tuple $(P_1, P_2, \ldots, P_n)$.

Unanimity

If each agent ranks x above y, then so does the social welfare function

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Universal Domain

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F is a total function.

Independence of Irrelevant Alternatives

The social relative ranking (higher, lower, or indifferent) of two alternatives x and y depends only the relative rankings of x and y for each individual.

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If for each $i \in \mathcal{A}$, xP_iy iff xP'_iy, then $xF(\vec{P})y$ iff $xF(\vec{P}')y$.

Borda does not satisfy IIA

# voters	3	2	2
	A	B	C
	B	C	A
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- The BC ranking is: A (8) $>$ B (7) $>$ C (6)

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- Add a new (undesirable) candidate X
- The new BC ranking is: C (13) $>$ B (12) $>$ A (11) $>$ X (6)

Dictatorship

There is an individual $d \in \mathcal{A}$ such that the society strictly prefers x over y whenever d strictly prefers x over y.

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There is a $d \in \mathcal{A}$ such that $xF(\vec{P})y$ whenever xP_dy.

Arrow's Theorem

Theorem (Arrow, 1951) Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.

Arrow's Theorem

K. Arrow. *Social Choice & Individual Values*. 1951.

Also, see

J. Geanakoplos. *Three Brief Proofs of Arrow's Impossibility Theorem*. Economic Theory, **26**, 2005.

A. Taylor. *Social Choice and The Mathematics of Manipulation*. Cambridge University Press, 2005.

W. Gaertner. *A Primer in Social Choice Theory*. Oxford University Press, 2006.

Recap: more than two candidates

- May's Theorem does not generalize (Condorcet Paradox)
- Many different procedures (Plurality, Plurality with runoff, Borda Count, Approval)
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- Different normative constraints on group decision methods are in conflict (Arrow's Theorem)

Next: More Group Rationality Constraints and Conclusions