Rationality

Lecture 5

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Accuracy and rationality are linked, they are not the same: a fool may hold a belief irrationally — as a result of a lucky guess or wishful thinking — yet it might happen to be correct. Conversely, a detective might hold a belief on the basis of a careful and exhaustive examination of all the evidence and yet the evidence may be misleading, and the belief may turn out to be wrong.

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Eric Schwitzgebel. *Belief*. In The Stanford Encyclopedia of Philosophy.

Franz Huber. *Formal Theories of Belief*. In The Stanford Encyclopedia of Philosophy.

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D. Christensen. *Putting Logic in its Place*. Oxford University Press.

Consistency Requirement

A rational agents (all-out) beliefs *should* (are rationally required to) be logically consistent.

Preface Paradox

D. Makinson. *The Paradox of the Preface*. Analysis, 25, 205 - 207, 1965.

I. Douven and J. Uffink. *The Preface Paradox Revisited*. Erkenntnis, 59, 389 - 420, 2003.

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But $\{s_1, \ldots, s_n, \neg(s_1 \wedge \cdots \wedge s_n)\}$ is logically inconsistent.

Preface Paradox

A philosopher who asserts "all of my present philosophical positions are correct" would be regarded as rash and over-confident

A philosopher who asserts "at least some of my present philosophical beliefs will turn out to be incorrect" is simply being sensible and honest.

Preface Paradox

1. each belief from the set $\{s_1, \ldots, s_n, s_{n+1}\}$ is rational
2. the set $\{s_1, \ldots, s_n, s_{n+1}\}$ of beliefs is rational.

1. does not necessarily imply 2.

Preface Paradox: The Problem

"The author of the book is being rational even though inconsistent. More than this: he is being rational even though he believes each of a certain collection of statements, which *he knows* are logically incompatible....this appears to present a living and everyday example of a situation which philosophers have commonly dismissed as absurd; that it is sometimes rational to hold incompatible beliefs."

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It is irrational to hold inconsistent beliefs at time t.

Diachronic: Rationality also involves the capacity that takes an agent from one mental state to another (either explicitly or implicitly through reasoning)

If S believes p and believes q at time t then S should (may/will) believe $p \wedge q$ at time $t' > t$.

Lottery Paradox

H. Kyburg. *Probability and the Logic of Rational Belief*. Wesleyan University Press, 1961.

I. Douven and T. Williamson. *Generalizing the Lottery Paradox*. British Journal of the Philosophy of Science, 57, 755 - 779, 2006.

G. Wheeler. *A Review of the Lottery Paradox*. Probability and Inference: Essays in honor of Henry E. Kyburg, Jr., College Publications, 2007.

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For each lottery ticket t_i $(i = 1, \ldots, 1000000)$, the agent believes that t_i will loose $B_A(\neg$'t_i will win')

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So, the conjunction $\bigwedge_{i=1}^{1000000}$ 't_i will not win' should be accepted. That is, the agent should rationally accept that *no lottery ticket will win*.

But, this is a fair lottery, so at least one ticket is *guaranteed* to win!

The Lottery Paradox

Kyburg: The following are inconsistent,

1. It is rational to accept a proposition that is very likely true,
2. It is not rational to accept a propositional that you are aware is inconsistent
3. It is rational to accept a proposition P and it is rational to accept another proposition P' then it is rational to accept $P \wedge P'$

Constraints on Graded Beliefs

Should a rational agent's graded beliefs satisfy the laws of probability?

J. Joyce. *Bayesianism*. in [HR].

The Dutch Book Argument

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Anyone whose beliefs violate the laws of probability is *practically irrational*.

F. P. Ramsey. *Truth and Probability*. 1931.

B. de Finetti. *La prévision: Ses lois logiques, ses sources subjectives*. 1937.

Alan Hájek. *Dutch Book Arguments*. Oxford Handbook of Rational and Social Choice, 2008.

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1. How do we make sense of decision making?
2. Evidence comes in a wide variety of types and strengths, and beliefs should be proportional to this evidence.

Graded Conditional Beliefs

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$C(X) = 1$ indicate **complete certainty in** X and $C(X) = 0$ indicates certainty that the proposition is false.

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1. Any adequate epistemology must recognize that opinions come in varying gradations of strength.
2. A person's graded beliefs can be represented using a set *Con* of confidence measures.
3. Facts about her beliefs correspond to properties shared by all elements of *Con*.

Reminder: Probability

A probability measure assigns to propositions an element of $[0,1]$ such that

Normalization $P(W) = 1$

Additivity $P(X \vee Y) = P(X) + P(Y)$ (also the countable version)

Conditional probability measure assigns to pairs of propositions an element of $[0,1]$ such that

Probability $P(\cdot \mid Y)$ is a probability measure for all Y

Conditional Normalization $P(Y \mid Y) = 1$

Conditioning $P(X \mid Y \wedge Z) \cdot P(Y \mid Z) = P(X \wedge Y \mid Z)$

Reminder: Probability

Logical Consequence: If X entails Y, then $P(X) \subseteq P(Y)$

Bayes' Theorem: $P(X \mid Y) = P(Y \mid X)\frac{P(X)}{P(Y)}$

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What is the rationale for this?

Next Week: Belief Change